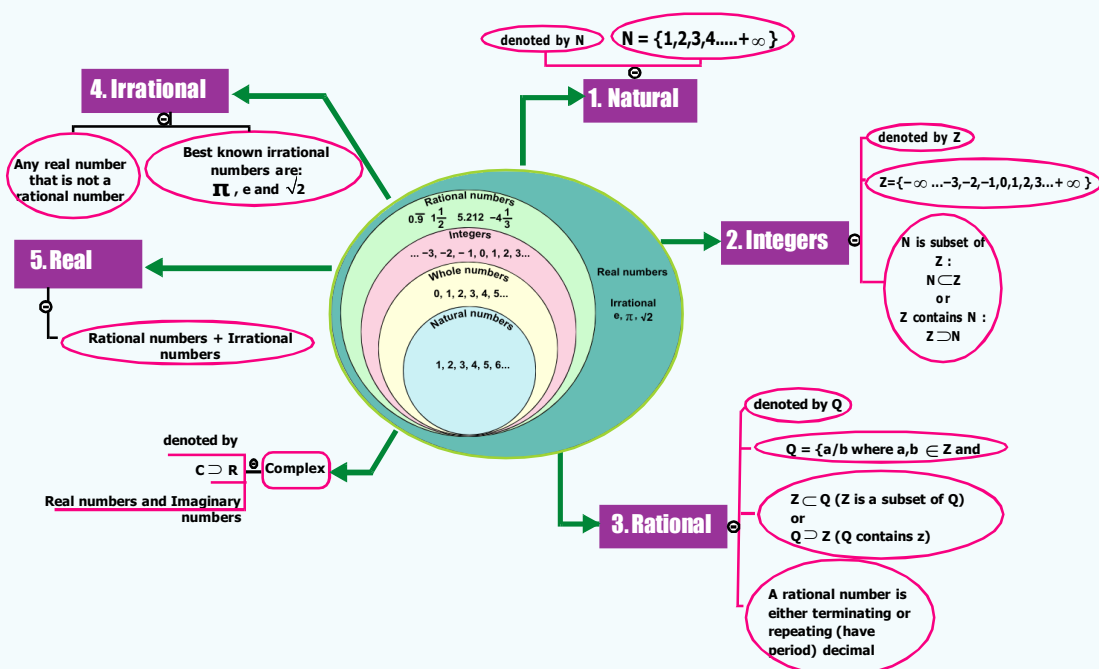


REAL NUMBERS

Richard Dedekind, in full name as Julius Wilhelm Richard Dedekind, German mathematician who developed a major redefinition of irrational numbers in terms of arithmetic concepts. Although not fully recognized in his lifetime, his treatment of the ideas of the infinite and of what constitutes a real number continues to influence modern mathematics. Dedekind developed the idea that both rational and irrational numbers could form a continuum (with no gaps) of real numbers, provided that the real numbers have a one-to-one relationship with points on a line.



CONCEPT MAP



CONCEPT 1.1

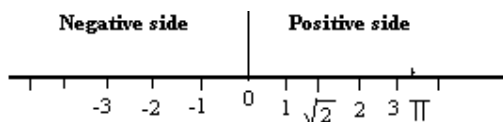
Real Number:

We have seen that if a number is an integer or it is expressible in a terminating or repeating decimal, then it is rational and if a number has a non-terminating and non-repeating decimal representation, then it is irrational. The totality of rationals and irrationals is called the set of real numbers. Real numbers are also defined as those numbers whose squares are non-negative. On the number line, each point corresponds to a unique real number and every real number occupies a unique point on the real line. It means that a number line is completely represented by real number. The union of the set of rational numbers Q and the set of irrational numbers Q' is known as the set of real numbers denoted by R .

Thus $R = Q \cup Q'$.

Clearly, $Q \cup Q' = R$ and $Q \cap Q' = \phi$ and $N \subset W \subset Z \subset Q \subset R, Q' \subset R$

Real number can be represented as points of a line. This line is called as a real line or number line.

**Rational Numbers:**

A number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number. The set of rational numbers is denoted by the capital letter ' Q ', and ' Q ' comes from the word 'Quotient'.

Example: $0, -1, \frac{3}{4}, \frac{5}{6}, \frac{-2}{3}, \frac{6}{7}, \dots$

Note: The word 'rational' is derived from the word 'ratio'.

Illustration: Find the sum of the rational numbers $\frac{-4}{9}$, $\frac{15}{12}$ and $\frac{-7}{18}$.

Solution: $\frac{-4}{9} + \frac{15}{12} + \frac{-7}{18} = \frac{-16+45-14}{36} = \frac{15}{36} = \frac{5}{12}$.

Properties of Addition of Rational numbers:

Closure property: Let $\frac{a}{b}$ and $\frac{c}{d}$ are two unique rational numbers such that $\frac{a}{b} + \frac{c}{d}$ is also a rational number.

Example: Let $\frac{-5}{12}, \frac{-1}{4}$ are any two rational numbers, $\left(\frac{-5}{12} + \frac{-1}{4}\right) = \frac{-5-3}{12} = \frac{-8}{12} = \frac{-2}{3}$, here $\frac{-2}{3}$ is also a rational number.

Commutative property: Let $\frac{a}{b}$ and $\frac{c}{d}$ are two unique rational numbers such that $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$.

Example: $\frac{3}{5} + \frac{11}{17} = \frac{51+55}{85} = \frac{106}{85}$ and $\frac{11}{17} + \frac{3}{5} = \frac{55+51}{85} = \frac{106}{85}$.
 $\therefore \frac{3}{5} + \frac{11}{17} = \frac{11}{17} + \frac{3}{5}$

Associative property: Let $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers such that $\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$

Example: Consider three rational numbers $\frac{1}{2}, \frac{3}{7}$ and $\frac{5}{9}$.
 Let $\left(\frac{1}{2} + \frac{3}{7}\right) + \frac{5}{9} = \left(\frac{7+6}{14}\right) + \frac{5}{9} = \frac{13}{14} + \frac{5}{9} = \frac{117+70}{126} = \frac{187}{126}$
 and $\frac{1}{2} + \left(\frac{3}{7} + \frac{5}{9}\right) = \frac{1}{2} + \frac{62}{63} = \frac{63+124}{126} = \frac{187}{126}$
 $\therefore \left(\frac{1}{2} + \frac{3}{7}\right) + \frac{5}{9} = \frac{1}{2} + \left(\frac{3}{7} + \frac{5}{9}\right)$

Additive Identity (Rule of 0):

$a + 0 = 0 + a = a$; '0' is called the additive identity of 'a'.

Example: '0' is a rational number such that the sum of any rational number and 0 is the rational number itself.

$$\Rightarrow \frac{7}{9} + 0 = \frac{7}{9} \text{ and } 0 + \frac{7}{9} = \frac{7}{9} \quad \therefore \frac{7}{9} + 0 = 0 + \frac{7}{9} = \frac{7}{9}$$

Illustration: Find the additive inverse of negative of a rational number $\frac{-23}{11}$?

Solution: Negative of a rational number $\frac{-23}{11}$ is $-\left(\frac{-23}{11}\right) = +\frac{23}{11}$.

The additive inverse of $\frac{23}{11}$ is $\frac{-23}{11}$.

\therefore The additive inverse of negative of a rational number $\frac{-23}{11}$ is $\frac{-23}{11}$.

Note:

The additive inverse of negative of any rational number is itself.



CLASSROOM DISCUSSION QUESTIONS

CDQ
1.1

1. What is a rational number?

- (A) A number expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$
- (B) A number expressed in decimal form
- (C) A number expressed in the form of $\frac{p}{q}$ where p and q are decimals
- (D) A number expressed in the form of $\frac{p}{q}$ where p and q are fractions

2. How are two rational numbers added together?

- (A) Multiply their numerators and denominators separately
- (B) Add their numerators and denominators separately
- (C) Add their numerators and divide by the common denominator
- (D) Subtract their denominators and divide by the common numerator

3. What property states that the sum of two rational numbers is also a rational number?

- (A) Closure property
- (B) Commutative property
- (C) Associative property
- (D) Distributive property

4. What is the additive identity of a rational number?

- (A) The sum of any rational number and 1
- (B) The sum of any rational number and 0
- (C) The product of any rational number and 1
- (D) The product of any rational number and 0

5. What is the additive inverse of a rational number?

- (A) The number itself
- (B) The negative of the number
- (C) The reciprocal of the number
- (D) The square of the number

6. How is the additive inverse of a rational number represented?

- (A) As the opposite sign of the number
- (B) As the reciprocal of the number
- (C) As the square root of the number
- (D) As the sum of the number and 1

7. Which property states that the order of addition of rational numbers does not affect the result?

- (A) Closure property
- (B) Commutative property
- (C) Associative property
- (D) Distributive property

8. What is the property called when the sum of two rational numbers equals zero?

- (A) Identity property
- (B) Zero property
- (C) Additive property
- (D) Inverse property

9. What does the term "closure property" refer to in rational numbers?

- (A) The sum of two rational numbers is always a rational number
- (B) The difference of two rational numbers is always a rational number
- (C) The product of two rational numbers is always a rational number
- (D) The quotient of two rational numbers is always a rational number

10. What is the set of rational numbers denoted by?

- (A) \mathbb{R}
- (B) \mathbb{P}
- (C) \mathbb{Q}
- (D) \mathbb{N}

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 A B C D	2 A B C D	3 A B C D	4 A B C D	5 A B C D
6 A B C D	7 A B C D	8 A B C D	9 A B C D	10 A B C D

CONCEPT 1.2

Subtraction of Rational Numbers:

Subtraction is the inverse process of addition. If $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers it follows $\frac{r}{s} - \frac{p}{q} = \frac{r}{s} + \left(-\frac{p}{q}\right)$

Illustration: Simplify: $\frac{3}{8} - \left(\frac{-2}{9}\right) + \left(\frac{-1}{36}\right)$.

Solution: $\frac{3}{8} - \left(\frac{-2}{9}\right) + \left(\frac{-1}{36}\right) = \frac{3}{8} + \frac{2}{9} - \frac{1}{36} = \frac{27+16+(-2)}{72} = \frac{41}{72}$.

Properties of Subtraction of Rational numbers:

Closure property: Let $\frac{a}{b}$ and $\frac{c}{d}$ are two unique rational numbers such that $\frac{a}{b} - \frac{c}{d}$ is also a rational number.

Example: Let $\frac{-5}{12}, \frac{-1}{4}$ are any two rational numbers, then $\left(\frac{-5}{12} - \left(\frac{-1}{4}\right)\right) = \left(\frac{-5}{12} + \frac{1}{4}\right) = \frac{-5+3}{12} = \frac{-2}{12} = \frac{-1}{6}$, here $\frac{-1}{6}$ is also a rational number.

Existence of right identity: In case of subtraction, any rational number $\frac{a}{b}$,

$$\frac{a}{b} - 0 = \frac{a}{b} \text{ but } 0 - \frac{a}{b} = -\frac{a}{b} \left(\text{not equal to } \frac{a}{b}\right).$$

Therefore, only the right identity exists for subtraction.

Note: The properties Commutative and Associative do not exist for rationals under subtraction.

Solved Problems:

1. What number should be added to $\frac{-5}{8}$ so as to get $\frac{-3}{2}$.

Solution: Let the number be x

$$x \text{ be added to } \frac{-5}{8}, \text{ we get } x + \frac{-5}{8} = \frac{-3}{2}$$

$$x = \frac{-3}{2} - \left(\frac{-5}{8}\right)$$

$$= \frac{-3}{2} + \frac{5}{8} = \frac{-3 \times 4 + 5}{8} = \frac{-7}{8}.$$

2. The sum of two rational numbers is -3 . If one of the numbers is $-\frac{10}{3}$, find the other number?

Solution: Let two numbers be x, y and let $y = -\frac{10}{3}$

$$x + y = -3 \Rightarrow x = -3 + \frac{10}{3} = \frac{-9 + 10}{3} = \frac{1}{3}$$

The other number is $\frac{1}{3}$.

Multiplication of rational numbers:

Let $\frac{a}{b}, \frac{c}{d}$ are two rational numbers such that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

Example: a) $\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$ b) $\frac{-1}{3} \times \frac{-5}{9} = \frac{5}{27}$

Properties of Multiplication of Rational numbers:

Closure Property: The product of two rational numbers is always a rational number. i.e., $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b} \times \frac{c}{d}\right)$ is also a rational number.

Example: $\left(-\frac{7}{11}\right) \times \left(-\frac{10}{11}\right) = \frac{70}{121} \in \mathbb{Q}$

Commutative property: Two rationals can be multiplied in any order.

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$

Example: $\left(-\frac{7}{11}\right) \times \left(-\frac{10}{11}\right) = \frac{70}{121} \Rightarrow \left(-\frac{10}{11}\right) \times \left(-\frac{7}{11}\right) = \frac{70}{121}$
 $\therefore \left(-\frac{7}{11}\right) \times \left(-\frac{10}{11}\right) = \left(-\frac{10}{11}\right) \times \left(-\frac{7}{11}\right)$

Note: We observe that the order of multiplication rationals does not change the sum. Then in such a case, we say that commutative property holds good under multiplication.

Associative Property: When multiply three (or) more rational numbers, we can be group them in any order.

For any three rationals $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$,

we have $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$

Example: $\left(\frac{1}{2} \times \frac{-3}{4}\right) \times \frac{5}{2} = \frac{1}{2} \times \left(\frac{-3}{4} \times \frac{5}{2}\right) \Rightarrow \left(\frac{-3}{8}\right) \times \frac{5}{2} = \left(\frac{1}{2} \times \frac{-15}{8}\right) \Rightarrow \frac{-15}{16} = \frac{-15}{16}$

Existence of multiplicative identity: For any rational number $\frac{a}{b}$, we have

$\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}$. Here, '1' is called multiplicative identity for rational numbers.

Example: a) $\frac{3}{5} \times 1 = 1 \times \frac{3}{5} = \frac{3}{5}$ b) $\frac{2}{11} \times 1 = 1 \times \frac{2}{11} = \frac{2}{11}$

Inverse Property: Every non-zero rational number $\frac{a}{b}$ has multiplicative inverse $\frac{b}{a}$, such that $\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$.

Example: a) $\frac{2}{3} \times \frac{3}{2} = \frac{3}{2} \times \frac{2}{3} = 1$. i.e., Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

b) $-\frac{5}{3} \times -\frac{3}{5} = -\frac{3}{5} \times -\frac{5}{3} = 1$. i.e., Reciprocal of $-\frac{5}{3}$ is $-\frac{3}{5}$

Distributive law of multiplication over addition: For any three rationals

$\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$, we have $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} \times \frac{e}{f}\right)$

Example: If $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ are three rational numbers, then

$$\Rightarrow \frac{1}{2} \times \left(\frac{2}{3} + \frac{3}{4}\right) = \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{3}{4}\right) \Rightarrow \frac{1}{2} \times \frac{17}{12} = \frac{2}{6} + \frac{3}{8} \Rightarrow \frac{17}{24} = \frac{8+9}{24} \Rightarrow \frac{17}{24} = \frac{17}{24}$$

Ex: If $\left[\frac{1}{12} + \left(\frac{-3}{4}\right) + \frac{7}{8}\right] \times \left[3\frac{2}{5} - \frac{7}{10} + \left(\frac{-2}{15}\right) - 10\frac{1}{30}\right] = x$ **then** $x + \frac{14}{9} =$

Sol: $\left[\frac{1}{12} + \left(\frac{-3}{4}\right) + \frac{7}{8}\right] \times \left[3\frac{2}{5} - \frac{7}{10} + \left(\frac{-2}{15}\right) - 10\frac{1}{30}\right] = \left(\frac{1}{12} - \frac{3}{4} + \frac{7}{8}\right) \times \left(\frac{17}{5} - \frac{7}{10} - \frac{2}{15} - \frac{301}{30}\right)$

$$= \left(\frac{2-18+21}{24}\right) \times \left(\frac{102-21-4-301}{30}\right) = \left(\frac{5}{24}\right) \times \left(\frac{-224}{30}\right) = \frac{-1}{3} \times \frac{28}{6} = \frac{-28}{18} = \frac{-14}{9}$$

But $\frac{-14}{9} = x$

$$\therefore x + \frac{14}{9} = \frac{-14}{9} + \frac{14}{9} = 0$$



CLASSROOM DISCUSSION QUESTIONS

CDQ
1.2

1. What is the result of subtracting

rational numbers $\frac{3}{8}$ and $\frac{1}{8}$ ((A) $\frac{27}{16}$ ((B) $\frac{16}{27}$ ((C) $\frac{16}{72}$ ((D) $\frac{27}{72}$

2. Which property holds true for the subtraction of rational numbers?

- (A) Closure property
(B) Commutative property
(C) Associative property
(D) Distributive property

3. What is the existence of the right identity for subtraction?

- (A) The sum of any rational number and 1
(B) The sum of any rational number and 0
(C) The product of any rational number and 1
(D) The product of any rational number and 0

4. Which property does not exist for rational numbers under subtraction?

- (A) Closure property
(B) Commutative property
(C) Associative property
(D) Identity property

5. What is the additive inverse of a rational number?

- (A) The number itself
(B) The negative of the number
(C) The reciprocal of the number
(D) The square of the number

6. What is the multiplicative identity for rational numbers?

- (A) 0 (B) 1 (C) -1 (D) 2

7. Which property states that the product of two rational numbers is always a rational number?

- (A) Closure property
(B) Commutative property
(C) Associative property
(D) Distributive property

8. What is the property called when two rationals can be multiplied in any order?

- (A) Closure property
(B) Commutative property
(C) Associative property
(D) Distributive property

9. What is the existence of multiplicative identity for rational numbers?

- (A) For any rational number $\frac{a}{b}$, we have $\frac{1}{b}$
(B) For any rational number $\frac{a}{b}$, we have $\frac{1}{a}$
(C) For any rational number $\frac{a}{b}$, we have $\frac{a}{a}$
(D) For any rational number $\frac{a}{b}$, we have $\frac{a}{b}$

10. What does the distributive law of multiplication over addition state for rational numbers?

- (A) The product of two rational numbers is always a rational number
(B) The sum of two rational numbers is always a rational number
(C) Rational numbers can be multiplied in any order
(D) Rational numbers can be grouped in any order for multiplication over addition

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1	A B C D	2	A B C D	3	A B C D	4	A B C D	5	A B C D
6	A B C D	7	A B C D	8	A B C D	9	A B C D	10	A B C D

CONCEPT 1.3**Division of rational numbers:**

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$ then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \text{reciprocal of } \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

Example: $-2\frac{1}{3} \div 3\frac{3}{4} = \frac{-7}{3} \div \frac{15}{4} = \frac{-7}{3} \times \frac{4}{15} = \frac{-7 \times 4}{3 \times 15} = \frac{-28}{45}$

Properties of Division:

Property 1: If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number. i.e., the set of all non-zero rational numbers is closed under division.

Example: $\frac{27}{16} \div \frac{9}{8} = \frac{27}{16} \times \frac{8}{9} = \frac{27 \times 8}{16 \times 9} = \frac{3 \times 1}{2 \times 1} = \frac{3}{2}$ is a rational number.

Property 2: For any rational number $\frac{a}{b}$, we have $\frac{a}{b} \div 1 = \frac{a}{b}$ and $\frac{a}{b} \div (-1) = -\frac{a}{b}$.

Example: $\frac{8}{21} \div 1 = \frac{8}{21}$, $\frac{8}{21} \div (-1) = \frac{8}{21} \div \frac{-1}{1} = \frac{8}{21} \times \frac{1}{-1} = \frac{8 \times 1}{21 \times -1} = \frac{8}{-21} = \frac{-8}{21}$.

Property 3: For every non-zero rational number $\frac{a}{b}$, we have

$$\text{i) } \frac{a}{b} \div \frac{a}{b} = 1 \quad \text{ii) } \frac{a}{b} \div \left(-\frac{a}{b}\right) = -1 \quad \text{iii) } -\frac{a}{b} \div \frac{a}{b} = -1$$

Ex: If 45 pants of equal size can be stitched with $\frac{195}{3}$ meters of cloth. Then the length of cloth required for each pant is

Sol: Given that 45 pants can be stitched from $\frac{195}{3}$ meters (of equal size) of cloth length of cloth required for each pant

$$= \frac{\left(\frac{195}{3}\right)}{45} = \frac{195}{3 \times 45} = \frac{65}{45} = \frac{13}{9} = 1\frac{4}{9} \text{ m}$$

Square root of a positive rational number:

Let 'a' be any positive rational number and we express $\sqrt{a} = b$, if and only if $b > 0$ and $b^2 = a$. The value of 'b' is called the positive square root of 'a'.

Example:

Find the value of $\sqrt{2}$ up to five decimal places.

Solution:

Let us find the value of $\sqrt{2}$ by long division method.

	1.4142135
1	2.00 00 00 00 00 00 00 00
	1
24	100
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
282841	383600
	282841
2828423	10075900
	8485269
28284265	159063100
	141421325
28284270	17611775

Step 1 : After 2, place decimal points.

Step 2 : After decimal points write 0's.

Step 3 : Group '0' is pairs and put a bar over them. (or) put dots on the numbers leaving digit after digit from right to left.

Step 4: First take down two numbers from left to right of the number. In this case it is 2.

Step 5: Consider the perfect square number which is very near to 2. Here it is 1.

Step 6: Then write down its square root namely 1 as divisor and also in the place of quotient.

Step 7: Find the product of 1 and 1 which is 1. So write down 1 below the number of 2 of the dividend.

Step 8: Subtract 1 from 2. Write down the remainder. So write down 1 below the number 2 of the dividend.

Step 9: Now bring down the digits up to the next bar or dot. They are 00. Write 00 beside 1. You get 100.

Step 10: Double the previous quotient namely 1. You get 2.

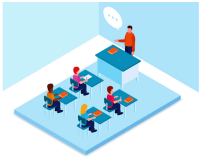
Step 11: Divide the two digits of the number from left to right. So obtained in step (9) by 2. You get 4:

Step 13: write 4 by the side of 2 to make the divisor 24.

Step 14: Put 4 by the side of 1 the quotient to get 1.4 .

Step 15: Multiply 24 by 4, the product is 96.

Step 16: Subtracting 96 from 100, you get 4 as the remainder. Since the remainder is zero the process of extracting square root of 2 is completed.



CLASSROOM DISCUSSION QUESTIONS

CDQ
1.3

- What is the result of dividing rational numbers $\frac{2}{3}$ and $\frac{7}{4}$?
a) $\frac{7}{15}$ b) $\frac{15}{7}$ c) $\frac{15}{28}$ d) $\frac{28}{15}$
- Which property states that the set of all non-zero rational numbers is closed under division?
a) Closure property
b) Commutative property
c) Associative property
d) Distributive property
- What is the value of $\frac{1}{8} \div \frac{1}{8}$?
a) 0 b) 1 c) -1 d) $\frac{1}{64}$
- For any rational number $\frac{a}{b}$, what is $\frac{-a}{b}$?
a) $\frac{1}{b}$ b) $\frac{-1}{b}$ c) $\frac{a}{1}$ d) $\frac{-a}{1}$
- What is the positive square root of $\frac{9}{16}$?
a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) $\frac{3}{16}$ d) $\frac{4}{9}$
- Which property of division states that $a \div a = 1$?
a) Property 1 b) Property 2
c) Property 3 d) Property 4
- What is the square root of 2, rounded to five decimal places?
a) 1.41422 b) 1.41421
c) 1.41420 d) 1.41423
- What is the result of dividing rational numbers $\frac{27}{16}$ and $\frac{3}{4}$?
a) $\frac{27}{8}$ b) $\frac{9}{4}$ c) $\frac{9}{8}$ d) $\frac{27}{4}$
- Which property of division states that $a \div a = 1$?
a) Property 1 b) Property 2
c) Property 3 d) Property 4
- What is the length of cloth required for each pant if 45 pants of equal size can be stitched from $\frac{195}{3}$ meters of cloth?
a) $\frac{1}{9}$ m b) $\frac{1}{4}$ m c) $\frac{1}{8}$ m d) $\frac{4}{9}$ m

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

- | | | | | |
|-----------|-----------|-----------|-----------|------------|
| 1 A B C D | 2 A B C D | 3 A B C D | 4 A B C D | 5 A B C D |
| 6 A B C D | 7 A B C D | 8 A B C D | 9 A B C D | 10 A B C D |

CONCEPT 1.4**Irrational Numbers:**

Some decimal fractions are neither terminating nor recurring decimals.

Example: i) 3.112123 1234 12345. . . ii) 1.732 050 807. . .

In the first example there appears to be a pattern or structure. It is neither terminating nor recurring decimal. The second one is also neither terminating nor recurring decimal. But there seems to be no pattern or structure. The decimal fractions cannot be expressed in the form of rational numbers, and therefore they are called irrational numbers.

Definition: A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. Thus, non-terminating, non-repeating decimals are irrational numbers. These are denoted by 'S' or 'Q'.

Examples of Irrational Numbers:**Type 1:**

- i) Clearly, 0.01001000100001.... is a non-terminating and non-repeating decimal and therefore, it is irrational,
- ii) 0.12112111211112 . . . , 0.54554555455554 . . . are irrationals and so on.

Type 2:

The 5th Century BC the Pythagorean in Greece, the follower of the famous mathematician and philosopher Pythagorean, proved that $\sqrt{2} = 1.4142135623731....$ is an irrational number. Later Theodorus of Cyrene shown that $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$, $\sqrt{15}$ and $\sqrt{17}$ are also irrational numbers.

Representing irrational numbers on number line:

Example: Represent each of the numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ on the real line.

Solution: Let X'OX be a horizontal line, taken as the X-axis and let 'O' be the origin and 'O' represent 0 .

- i) Take OA = 1 unit and draw OB perpendicular to \overline{OA} at O. Take OB = 1 unit. Draw \overline{BA} . (The Pythagoras property of a right angled triangle states that "The square on the hypotenuse is equal to the sum of the squares on the other two sides"). From the right angled $\Delta^{\text{c}} BOA$, $AB^2 = BO^2 + OA^2$

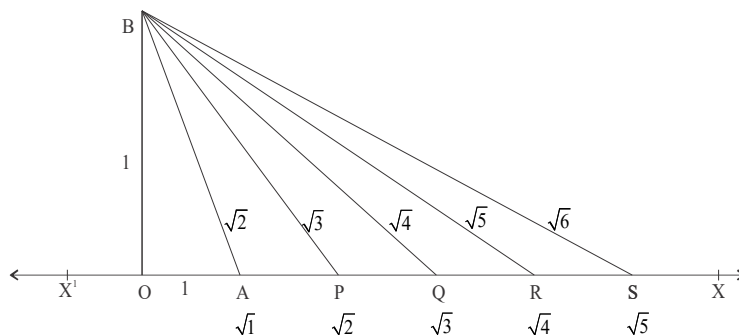
$$\Rightarrow AB^2 = 1^2 + 1^2 \quad \therefore AB^2 = 2 \quad \Rightarrow AB = \sqrt{2}.$$

- ii) With 'O' as centre and radius equal to AB draw an arc, meeting OX at P.

Draw OB perpendicular to \overline{OP} at O. Take OB = 1 unit. Draw \overline{BP} . Then, OP = AB = $\sqrt{2}$ units. Thus, the point 'P' represents $\sqrt{2}$ on the real line.

From the right angled Δ^{e} BOP, $BP^2 = OB^2 + OP^2 = 1^2 + (\sqrt{2})^2 = 1 + 2 = 3$

$$\therefore BP^2 = 3 \Rightarrow BP = \sqrt{3}$$



- iii) With 'O' as centre and radius equal to BP describe an arc, meeting OX at Q. Draw OB perpendicular to \overline{OQ} at O. Take OB = 1 unit. Draw \overline{BQ} . Then OQ = BP = $\sqrt{3}$ units. Thus, the point 'Q' represents $\sqrt{3}$ on the real line.

From the right angled Δ^{e} BOQ, $BQ^2 = OB^2 + OQ^2 = (\sqrt{3})^2 + 1^2$

$$\therefore BQ^2 = 4 \Rightarrow BQ = \sqrt{4} = 2 \text{ units}$$

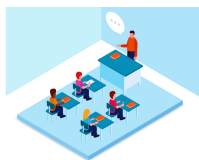
- iv) With 'O' as centre and radius equal to BQ describe an arc, meeting OX at R. Draw OB perpendicular to \overline{OR} at O. Take OB = 1 unit. Draw \overline{BR} . Then OR = BQ = $\sqrt{4}$ units. Thus, the point 'R' represents ' $\sqrt{4}$ ' on the real line.

From the right angled Δ^{e} BOR, $BR^2 = OB^2 + OR^2 = (\sqrt{4})^2 + 1^2$

$$\therefore BR^2 = 5 \Rightarrow BR = \sqrt{5} \text{ units} \text{ Then } OS = BR = \sqrt{5} \text{ units.}$$

Thus, the point 'S' represents $\sqrt{5}$ on the real line.

Hence, the points P, Q, S represent the numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ respectively.



CLASSROOM DISCUSSION QUESTIONS

CDQ
1.4

1. **What is the definition of an irrational number?**
 - (A) A number expressed as a terminating decimal
 - (B) A number expressed as a repeating decimal
 - (C) A number expressed as a non-terminating, non-repeating decimal
 - (D) A number expressed as an integer
2. **Which of the following decimal fractions is an irrational number?**
 - (A) 0.123123123...
 - (B) 0.333333...
 - (C) 0.545454...
 - (D) 0.666666...
3. **Who proved that $\sqrt{2}$ is an irrational number?**
 - (A) Pythagoras
 - (B) Theodorus of Cyrene
 - (C) Euclid
 - (D) Archimedes
4. **What is the value of $\sqrt{2}$ approximately equal to?**
 - (A) 1.414
 - (B) 2.414
 - (C) 1.732
 - (D) 3.142
5. **What type of decimal is 0.01001000100001...?**
 - (A) Terminating
 - (B) Repeating
 - (C) Irrational
 - (D) Rational
6. **Which point on the number line represents the number 4?**
 - (A) P (B) Q (C) R (D) S
7. **What does π approximately equal to?**
 - (A) 3.142857
 - (B) 3.141592653589793238
 - (C) 3.14
 - (D) 3.145
8. **Is π a rational number?**
 - (A) Yes (B) No
 - (C) (D)
9. **Which of the following statements about π is true?**
 - (A) π is a repeating decimal.
 - (B) π is a terminating decimal.
 - (C) π is an irrational number.
 - (D) π is a rational number.
10. **What is celebrated on March 14th, known as Pi Day?**
 - (A) The birthday of Albert Einstein
 - (B) The discovery of irrational numbers
 - (C) The value of π
 - (D) The value of π as 3.14

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

- | | | | | |
|-----------|-----------|-----------|-----------|------------|
| 1 A B C D | 2 A B C D | 3 A B C D | 4 A B C D | 5 A B C D |
| 6 A B C D | 7 A B C D | 8 A B C D | 9 A B C D | 10 A B C D |

CONCEPT 1.5

Density Property: There are infinitely many irrational numbers between any two rational numbers. This property is known as denseness property or density property.

Example: Find any two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$.

Solution: We know that $\frac{1}{5} = 0.20$, $\frac{2}{7} = 0.\overline{285714}$.

To find two irrational numbers between $\frac{1}{5}$ and $\frac{2}{7}$, we need to look at the decimal form of the two numbers and then proceed. We can find infinitely many such irrational numbers.

0.201201120111..., 0.24114111411114..., 0.25231617181912..., 0.267812147512... are irrational numbers.

Note: If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then \sqrt{ab} is an irrational number lying between a and b .

Example: Find two irrational numbers between 2 and 3.

Solution: An irrational number between 2 and 3 is $\sqrt{2 \times 3} = \sqrt{6}$.

Similarly an irrational number between 2 and $\sqrt{6} = \sqrt{2 \times \sqrt{6}} = \sqrt{2\sqrt{6}}$

\therefore Required numbers are $\sqrt{6}$ and $\sqrt{2\sqrt{6}}$ lie between 2 and 3.

Example: Find two irrationals between $\sqrt{3}$ and $\sqrt{5}$.

Solution: An irrational number between $\sqrt{3}$ and $\sqrt{5}$ is $\sqrt{\sqrt{15}} = 15^{\frac{1}{4}}$

Irrational number between $\sqrt{3}$ and $\sqrt[4]{15}$ is

$$\sqrt{(3^{\frac{1}{2}})(15)^{\frac{1}{4}}} = (3^2)^{\frac{1}{8}} \times (15)^{\frac{1}{8}} = \sqrt[8]{135}$$

\therefore Required numbers are $\sqrt[4]{15}$, $\sqrt[8]{135}$ lie between $\sqrt{3}$ and $\sqrt{5}$.

Properties of Irrational Numbers:

1. The sum of two irrationals is either rational or irrational.

Example:

i) $(3 + \sqrt{5})$ and $(9 - \sqrt{5})$ are irrationals but $(3 + \sqrt{5}) + (9 - \sqrt{5}) = 12$ is rational.

ii) $(7 + \sqrt{3})$ and $(-2 - 4\sqrt{3})$ are irrationals and $(7 + \sqrt{3}) + (-2 - 4\sqrt{3}) = 5 - 3\sqrt{3}$ is irrational.

2. The difference of two irrationals is either rational or irrational.

Example: i) $(8 + \sqrt{7}) - (5 + \sqrt{7}) = 3$ is rational

ii) $(3 - \sqrt{5}) - (2 + 3\sqrt{5}) = 1 - 4\sqrt{5}$ is irrational.

3. The product of two irrationals is either rational or irrational.

Example: i) $(3 + \sqrt{5}) \times (3 - \sqrt{5}) = 3^2 - (\sqrt{5})^2 = 9 - 5 = 4$, is a rational

ii) $(7 + \sqrt{3}) \times (3 + \sqrt{2}) = 7 \times 3 + 7 \times \sqrt{2} + \sqrt{3} \times 3 + \sqrt{3} \times \sqrt{2}$
 $= 21 + 7\sqrt{2} + 3\sqrt{3} + \sqrt{6}$ is irrational

4. The quotient of two irrationals is either rational or irrational.

Example: i) $\frac{7\sqrt{3}}{4\sqrt{3}} = \frac{7}{4}$ which is rational, though each of $7\sqrt{3}$ and $4\sqrt{3}$ are irrationals.

ii) $\frac{5\sqrt{2}}{2\sqrt{3}} = \frac{5}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{5}{2} \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5}{6} \sqrt{6}$ which is irrational.

5. Irrational numbers satisfy the commutative, associative and distributive laws.

Note:

i) If a is rational & \sqrt{b} is irrational, then $(a \pm \sqrt{b})$, $(a\sqrt{b})$ and $\frac{a}{\sqrt{b}}$ are irrationals.

ii) If a is a positive integer, which is not a perfect square then \sqrt{a} is irrational.

iii) If a is a positive integer which is not a perfect cube, then $\sqrt[3]{a}$ is irrational.

iv) $\sqrt{-a}$ is not a real number, because $\sqrt{-a} = \sqrt{-1} \times \sqrt{a}$, $\sqrt{-1} \notin \mathbb{R}$.

Know about π : $\pi = 3.141592653589793238 \dots$

Example: Is π a rational number?

Solution: π is the ratio of the circumference of a circle to the length of the diameter. The value of π is approximately equal to $\frac{22}{7}$ but not exactly.

$\pi = 3.14159265\dots$ (π is an irrational number)

$\frac{22}{7} = 3.\overline{142857}$ ($\frac{22}{7}$ is a rational number)

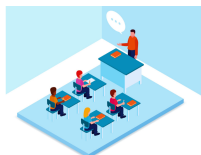
A closer approximation to π is $\frac{355}{113}$ which equals to 3.141592...

The decimal expansion of π is non-terminating non-recurring.

So π is an irrational number.

Note: We often take $\frac{22}{7}$ as an approximate value of π , but $\pi \neq \frac{22}{7}$. We celebrate

March 14th as π day since it is 3.14 and time 1:59 (as $\pi = 3.14159$) what a coincidence, Albert Einstein was born on March 14th, 1879.



CLASSROOM DISCUSSION QUESTIONS

CDQ
1.5

- What is the density property of real numbers?**
 - There are infinitely many rational numbers between any two irrational numbers.
 - There are infinitely many irrational numbers between any two rational numbers.
 - There are only finitely many irrational numbers between any two rational numbers.
 - There are only finitely many rational numbers between any two irrational numbers.
- How many irrational numbers are there between 1 and $2/7$?**
 - 0
 - 1
 - 2
 - Infinitely many
- Which property states that the sum of two irrationals is either rational or irrational?**
 - Property 1
 - Property 2
 - Property 3
 - Property 4
- What is the product of $\sqrt{3} + \sqrt{5}$ and $\sqrt{3} - \sqrt{5}$?**
 - Rational
 - Irrational
 - Zero
 - Undefined
- If α is a positive integer which is not a perfect square, then $\sqrt{\alpha}$ is:**
 - Rational
 - Irrational
 - Undefined
 - Negative
- What is the set of real numbers denoted by \mathbb{R} ?**
 - Set of rational numbers
 - Set of irrational numbers
 - Set of complex numbers
 - Union of rational and irrational numbers
- What is the union of the set of rational numbers \mathbb{Q} and the set of irrational numbers \mathbb{Q}' ?**
 - Set of integers
 - Set of real numbers
 - Set of natural numbers
 - Set of whole numbers
- What does \mathbb{R} represent?**
 - Set of rational numbers
 - Set of irrational numbers
 - Set of real numbers
 - Set of complex numbers
- Is $-\alpha$ a real number?**
 - Yes
 - No
 - (C)
 - (D)
- How are real numbers represented on a line?**
 - By points on a circle
 - By points on a plane
 - By points on a parabola
 - By points on a real line or number line

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

- | | | | | |
|-----------|-----------|-----------|-----------|------------|
| 1 A B C D | 2 A B C D | 3 A B C D | 4 A B C D | 5 A B C D |
| 6 A B C D | 7 A B C D | 8 A B C D | 9 A B C D | 10 A B C D |

1. **Properties of Rational numbers:**

Let $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers such that

- a) Additive Identity: $a + 0 = 0 + a = a$; '0' is called the additive identity of 'a'.
 - b) Additive inverse: $\frac{p}{q} + \frac{r}{s} = 0$, then $\frac{p}{q}$ is called the additive inverse or negative of $\frac{r}{s}$.
 - c) Multiplicative identity: $\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}$. Here, '1' is called multiplicative identity.
 - d) Inverse Property: $\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$. Here rational number $\frac{a}{b}$ is multiplicative inverse $\frac{b}{a}$.
2. The rational number $\frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.
 3. Zero has no reciprocal.
 4. Reciprocal of '1' is '1' and reciprocal of '-1' is '-1'.
 5. Let 'a' be any positive rational number and we express $\sqrt{a} = b$ if and only if $b > 0$ and $b^2 = a$. The value of 'b' is called the positive square root of 'a'.
 6. A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number and is denoted by 'S' or 'Q'.
 7. Non-terminating, non-repeating decimals are irrational numbers.
 8. The decimal expansion of π ($\pi = 3.14159 \dots$) is non-terminating non-recurring and it is an irrational number.
 9. There are infinitely many irrational numbers between any two rational numbers. This property is known as denseness property or density property.
 10. If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then \sqrt{ab} is an irrational number lying between a and b.
 11. Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.
 12. Sum/Difference/Product/Quotient of two irrationals need not be an irrational.
 13. A number whose square is non-negative is called a real number.
 14. Real numbers are denoted by R, thus $R = Q \cup Q'$.
 15. $N \subset W \subset Z \subset Q \subset R$, $Q' \subset R$.

ADVANCED WORKSHEET



Single Correct Answer Type (S.C.A.T)

1. A fraction is a rational number.

- (A) Never a rational number
- (B) Yes, it is an rational number
- (C) Only a fraction
- (D) It is an irrational number

2. If $\frac{a}{b}$ is the additive inverse of $\frac{c}{d}$

then $\frac{a}{b} + \frac{c}{d} = \underline{\hspace{2cm}}$.

- (A) 0
- (B) $\frac{a}{b} \times \frac{c}{d}$

(C) $\frac{a+c}{b+d}$

(D) 1

3. If $p^2 = q$, where $p > 0$ then which of the following is true:

- (A) p is positive square root of q
- (B) p is square of q
- (C) q is square of p^2
- (D) q is positive square root of p

4. A non terminating decimal from the following is:

- | | |
|---------------------|---------------------|
| (A) $\frac{7}{125}$ | (B) $\frac{4}{200}$ |
| (C) $\frac{5}{27}$ | (D) $\frac{3}{64}$ |

5. Which of the following is irrational?

(A) $\sqrt{625}$

(B) $\sqrt{1024}$

(C) $\sqrt{23}$

(D) $\sqrt{144}$

6. π day celebrates on:

- (A) November 14th
- (B) April 14th
- (C) March 14th
- (D) August 14th

7. The irrational number between 3 and 7:

(A) $\sqrt{63}$

(B) $\sqrt{2}$

(C) $\sqrt{50}$

(D) $\sqrt{21}$

8. $\pi + 7$ is a ____.

- (A) Rational
- (B) Irrational
- (C) Can't be determined
- (D) None of the above

9. Numbers which cannot be expressed in the form $\frac{p}{q}$, $p, q \in$

\mathbb{Z} , $q \neq 0$ are called:

- (A) Rational
- (B) Irrational
- (C) Fractional
- (D) Decimal

10. Which of the following is/are rational number(s)?

- (A) $(3 + \sqrt{5})$
- (B) $(-1 + \sqrt{3})$
- (C) $5\sqrt{16}$
- (D) $-\sqrt{7}$

11. Which of the following is a/an rational number?

- (A) $\frac{\sqrt{72}}{\sqrt{2}}$
- (B) $\frac{3}{\sqrt{2}}$
- (C) $5\sqrt{6}$
- (D) $-\sqrt{7}$

12. Which of the following is a/an irrational number?

- (A) 0.15
- (B) 0.01516
- (C) $0.\overline{1516}$
- (D) 0.5015001500015...

13. Which of the following is an irrational number?

- (A) $\sqrt{4}$
- (B) $\sqrt{5}$
- (C) 1.5
- (D) $1.\bar{5}$

14. Which one of the following is not an irrational?

- (A) $\sqrt{2}$
- (B) $\sqrt{3}$
- (C) $\sqrt{4}$
- (D) $\sqrt{5}$

15. The collection of rationals and irrationals is called ____.

- (A) Real numbers
- (B) Integers
- (C) Natural numbers
- (D) Rational numbers

16. An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is:

- (A) $6^{\frac{1}{4}}$
- (B) $6^{\frac{1}{6}}$
- (C) $6^{\frac{1}{8}}$
- (D) $12^{\frac{1}{8}}$

17. Sum of $2+\sqrt{5}$ and $\sqrt{5}-2$ is a:

- (A) Rational number
- (B) Irrational number
- (C) Fraction
- (D) None of these

18. Simplify: $\frac{2}{3} - \frac{4}{5} + \frac{7}{15} - \frac{11}{20}$.

- (A) $\frac{-7}{30}$
- (B) $\frac{-4}{15}$
- (C) $\frac{-1}{5}$
- (D) $\frac{-13}{60}$

19. On one day Auto driver spent Rs. $\frac{500}{3}$ on repairs, Rs. $\frac{300}{2}$ on New seat cover, and Rs. $\frac{1500}{4}$ on oil, then total how much amount spent:

- (A) Rs. $\frac{7300}{12}$
- (B) Rs. $\frac{6800}{11}$
- (C) Rs. $\frac{8300}{12}$
- (D) Rs. 5000

20. The dimensional frame of TV display screen is rectangular in shape, whose length and width are $\frac{300}{5}$ inches and $\frac{220}{5}$ inches then the perimeter of TV is:

- (A) 202 inches
- (B) 294 inches
- (C) 252 inches
- (D) 208 inches

21. What should be added to $\left(\frac{4}{5} + \frac{3}{7} + \frac{5}{3}\right)$ to get $\frac{3}{2}$?

- (A) $\frac{-293}{210}$
- (B) $\frac{-107}{210}$
- (C) $\frac{-107}{110}$
- (D) $\frac{-207}{200}$

22. A Basket contains three types of fruits weighing $\frac{582}{3}$ kg in all. If

$\frac{585}{5}$ kg these be grapes, $\frac{585}{15}$ kg of these be oranges, the rest be mangoes, Then the weight of mangoes in basket is:

- (A) 36 kg
- (B) 44 kg
- (C) 35 kg
- (D) 38 kg

23. If $\left[\frac{1}{12} + \left(\frac{-3}{4} \right) + \frac{7}{8} \right] \times$

$\left[3\frac{2}{5} - \frac{7}{10} + \left(\frac{-2}{15} \right) - 10\frac{1}{30} \right] = x$ then

$x + \frac{14}{9} = \underline{\hspace{2cm}}$.

- (A) 1
- (B) -1
- (C) 2
- (D) 0

24. The value of $\sqrt{8}$ upto 3 decimal places is .

- (A) 2.428
- (B) 2.828
- (C) 3.828
- (D) 8

25. $(2 + \sqrt{2})(2 - \sqrt{2})$ is a number.

- (A) Rational
- (B) Irrational
- (C) Can't be determined
- (D) None of the above

26. If sum of two rational numbers is -8 and one of the rational

numbers is $-\frac{17}{9}$, then the other one is .

- (A) $-\frac{50}{9}$
- (B) $\frac{50}{9}$
- (C) $-\frac{55}{9}$
- (D) $-\frac{65}{9}$

27. If $17\frac{2}{3}$ of 180 + $\frac{1}{4}$ of 480 = x, then the value of x is .

- (A) 3180
- (B) 3420
- (C) 3200
- (D) 3300

28. If product of two numbers is $25\frac{3}{8}$ and one of them is $15\frac{9}{40}$, then other number is .

- (A) $\frac{2}{3}$
- (B) $1\frac{2}{3}$
- (C) $5\frac{2}{3}$
- (D) $\frac{9}{7}$

29. If $15\frac{2}{3} \times 3\frac{1}{6} + 6\frac{1}{3} = 11\frac{7}{8} + x$, then the value of x is .

- (A) $39\frac{5}{9}$
- (B) $137\frac{4}{9}$
- (C) $29\frac{7}{9}$
- (D) $44\frac{5}{72}$

30. How many numbers in the list are irrationals

$$\sqrt{3}, \pi, \frac{1}{3}, 0, \sqrt[5]{2}, \frac{22}{7}, \sqrt{36} ?$$

- (A) 3
- (B) 4
- (C) 5
- (D) 6

31. Given $\sqrt{2} = 1.414$ then value of $\sqrt{8}$ is ____.

- (A) 2.848
- (B) 3.464
- (C) 3.644
- (D) 2.828

32. Given $\sqrt{3} = 1.732$ then value of $\sqrt{75}$ is ____.

- (A) 8.65
- (B) 8.67
- (C) 8.66
- (D) 8.661

33. Multiply: $\sqrt{14}$ by $\sqrt{8}$.

- (A) $4\sqrt{7}$
- (B) $2\sqrt{14}$
- (C) 24
- (D) $\sqrt{28}$

34. The multiplicative inverse of

$$-8\frac{4}{9} \text{ is ____}.$$

- (A) $6^{\frac{1}{2}}$
- (B) $68/9$
- (C) $-9/76$
- (D) $-9/68$

35. If $\sqrt{5} = 2.23607$, then the value of $\sqrt{20}$ is ____.

- (A) 4.47214
- (B) 5.47214
- (C) 4.27214
- (D) 5.42724



Multi Correct Question (M.C.Q)

36. Which of following are not reciprocals of rational numbers lying between 1 and 4?

- (A) $2/9$
- (B) $5/17$
- (C) $4/3$
- (D) $3/4$

37. Which of the following are false?

- (A) $\frac{1}{50}$ can be written as terminating decimal
- (B) $3\frac{5}{7}$ can be written as terminating decimal
- (C) $\frac{1}{22}$ can be written as terminating decimal
- (D) $\frac{2}{15}$ can be written as terminating decimal

38. If the sum of additive inverse of $\frac{4}{72}$ and additive inverse of $\frac{215}{18}$ is x , then the value of x is ____.

- (A) 17
- (B) -12
- (C) Additive inverse of 12
- (D) Additive inverse of -17

39. If $a = 1000$, $b = 5$, $c = 25$, $d = 125$, then $\frac{a}{b} + \frac{a}{c} + \frac{a}{d}$ is ____.

- (A) 0.248
- (B) 248
- (C) Less than 250
- (D) Less than 248

40. Which of the following are rational numbers?

- (A) $\sqrt{5}$
- (B) $\sqrt{25}$
- (C) $\sqrt{7}$
- (D) $\sqrt{1.44}$

41. Which of the following is/are a correct statement?

- (A) In $b^2 = a$, $b > 0$, then 'b' is positive square root of 'a'.
- (B) π is an irrational number
- (C) Irrational numbers cannot be represented by points on the number line
- (D) Irrational number can be expressed in the form of $\frac{p}{q}$, $q \neq 0$.

42. $\sqrt{\left(a + \frac{1}{2}\right)^2 + \frac{3}{4}}$ is irrational if 'a' is

- (A) 0
- (B) 1
- (C) 2
- (D) -1

43. The irrational numbers lie between $\sqrt{2}$ and $\sqrt{7}$ is /are:

- (A) $(14)^{\frac{1}{4}}$
- (B) $(35)^{\frac{1}{4}}$
- (C) $(56)^{\frac{1}{8}}$
- (D) $(126)^{\frac{1}{4}}$

44. Which of the following is / are correct?

- (A) Product of two rationals is a rational
- (B) Product of two irrationals is an irrational
- (C) Sum of a rational and an irrational is an irrational
- (D) Product of a rational and an irrational is a rational.

Comprehension Passage (C.P.T)

PASSAGE - I

If $A = \frac{5}{6} - \left(\frac{1}{2} + \frac{1}{3}\right)$ and $B = 1\frac{3}{4} - \left(\frac{1}{2} + \frac{1}{4}\right)$

then:

45. The value of 'A' is:

- (A) 0
- (B) 2
- (C) 1
- (D) 3

46. The value of 'B' is:

- (A) 0
- (B) 2
- (C) 1
- (D) 3

47. B - A = ____.

- (A) 1
- (B) 3
- (C) 4
- (D) 6

PASSAGE - II

If a, b are positive rational numbers and ab is not a perfect square then \sqrt{ab} is irrational number between a and b.

Then

48. $\sqrt{15}$ is irrational number between:

- (A) 3 and 5
- (B) 13 and 17
- (C) 4 and 6
- (D) 4 and 8

49. If p is one of irrational number between 13 and 15 then

$$\sqrt{\frac{p^2 + 5}{2}} = _.$$

- (A) 15
- (B) 20
- (C) 25
- (D) 10

50. If x is irrational between 5, 10 and y is irrational number between 2 and 25 then

$$\frac{x^2 + y^2}{10} = _.$$

- (A) 10
- (B) 3
- (C) 2
- (D) 4



Matrix Matching Type (M.M.T.)

I. Column - I

51. Additive inverse of $\frac{-15}{-11}$ is

52. Additive identity of $\frac{-8}{9}$ is

53. If $x = \frac{-8}{9}$, then $-(-x) =$

54. If $x = \frac{-3}{4}$ and $y = \frac{-6}{7}$, then $(x+y) =$

Column-II

- (A) $\frac{-8}{9}$
- (B) $\frac{15}{11}$
- (C) $\frac{-45}{28}$
- (D) $\frac{-15}{11}$
- (E) $\frac{8}{9}$

II. Column - I

55. p is rational, q is irrational then

$p+q$, $p-q$, pq , $\frac{p}{q}$ are

56. $\pi \geq 4$

57. All real numbers are rational

58. Approximate value of $\frac{\sqrt{33}}{2}$

Column - II

- (A) 2.87
- (B) True
- (C) Irrational
- (D) 3.87
- (E) false

Assertion Reason Type (A.R.T.)

(A) Both Assertion(A) and Reason(R) are correct and reason(R) is the correct explanation of assertion(A).

(B) Both Assertion(A) and Reason(R) are correct but reason(R) is not the correct explanation of assertion(A).

(C) Assertion(A) is correct but Reason(R) is incorrect.

(D) Assertion(A) is incorrect but Reason(R) is correct.

59. **Assertion (A):** $2 + \sqrt{6}$ is an irrational number.

Reason (R): Sum of a rational number and an irrational number is always an irrational.

60. **Assertion (A):** If $a, b, c \in \mathbb{Q}$, then $a - (b - c) = (a - b) - c$.

Reason(R): Subtraction does not exist under associative property.

Integer Type Question (I.T.Q.)

61. If $\frac{2}{3} - \frac{4}{5} + \frac{7}{15} - \frac{11}{20} = \frac{-13}{12 \times k}$ then $k =$ ____.

62. If 45 pants of equal size can be stitched from $\frac{195}{3}$ meters of cloth and If the length of cloth required for each pant is $1\frac{p}{9}$ meters then $p =$ ____.

63. If $1.\overline{46} = \frac{145}{11 \times q}$ then $q =$ ____.

64. If $(77)^{\frac{1}{n}}$ is a rational number between 7 and 11 then least possible value for $n =$ ____.

Figure Question (P.Q.)

65. **Decimal Expansion of π is** ____.
[NTSE]

- (A) A whole number
- (B) Terminating
- (C) Non-terminating but repeating
- (D) Non-terminating non-repeating

66. The sum of rational and irrational number is: [NTSE]

- (A) Rational
- (B) Irrational
- (C) Zero
- (D) Integers

67. Which of the following can be expressed as the sum of square of two positive integers. as well as three positive integers?

[NTSE]

- (A) 75
- (B) 192
- (C) 250
- (D) 100

68. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number?

[NCERT Exemplar]

- (A) Yes
- (B) No
- (C) Can't say
- (D) Sometimes

69. Consider the following statements:

[NCERT Exemplar]

- (i) $\frac{\sqrt{2}}{3}$ is a rational number.
- (ii) There are infinitely many integers between any two integers.
- (iii) Number of rational numbers between 15 and 18 is finite.
- (iv) There are rational numbers which cannot be written in the form p/q , $q \neq 0$, p, q both are integers Which of the following is true?

- (A) Only statement (i) is true
- (B) All statement are false except (iv)
- (C) All statement are false
- (D) Only statement (iii) is false

NOTES

[illegible]