Chapter 1

IRRATIONAL NUMBERS AND SURDS

Hippasus of Metapontum was a Pythagorean philosopher. Little is known about his life or his beliefs, but he is sometimes credited with the discovery of the existence of irrational numbers. The discovery of irrationality is not specifically ascribed to Hippasus by any ancient writer. Some modern scholars though have suggested that he discovered the irrationality of $\sqrt{2}$, which it is believed was discovered around the time that he lived.

CONCEPT MAP

IRRATIONALS AND SURDS

- Introduction to Irrational Number
- Irrationals on Number line
- Properties of Irrational Numbers
- Introduction to Surds
- Types of surds
- Convertion of surds
- Operations on Surds
- Rationalisation of Surds
- Conjugate Surds

CONCEPT 1.1

Definition: A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number.

Thus, non-terminating, non-repeating decimals are irrational numbers. These are denoted by 'S' or 'Q' '.

Irrational Numbers:

Some decimal numbers are neither terminating nor recurring decimals.

Example: i) 3.1121231234 12345. . . ii) 1.732050 807. . .

In the first example, there appears to be a pattern of structure. It is neither terminating nor recurring decimal. The second one is also neither terminating nor recurring decimal. But there seems to be no pattern or structure. The decimal fractions cannot be expressed in the form of rational numbers, and therefore they are called irrational numbers.

Examples of Irrational Numbers:

Type 1: i) Clearly, 0.01001000100001. . . is a non-terminating and non-repeating decimal and therefore, it is irrational,

ii) 0.12112111211112 . . ., 0.54554555455554 . . . are irrationals.

Type 2: The 5^{th} Century BC the Pythagorean in Greece, the follower of the famous mathematician and philosopher Pythagorean, proved that $\sqrt{2}$ = 1.4142135623731 . . . is an irrational number. Later Theodorus of Cyrene showed that $\sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{10}, \sqrt{11}, \sqrt{12}, \sqrt{13}, \sqrt{14}, \sqrt{15}$ and $\sqrt{17}$ are also irrational numbers.

Type 3: The number 'e'(Euler's number) is an irrational . Its value is 2.71828182845. . . (It is called Napier constant..... also).

i.e., $2 < e < 3$

Know about π:

π = 3.14159265358979323 8 . . .

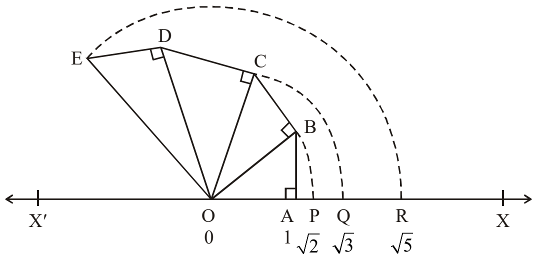
The decimal expansion of π is non-terminating non-recurring. So π is an irrational number. Note that, we often take $\frac{22}{7}$ as an approximate value of π, but $\pi \neq \frac{22}{7}$.

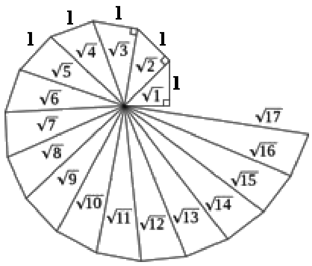
We celebrate March 14th as π day, since it is 3.14 and at time 1:59 (as π = 3.14159). What a coincidence, Albert Einstein was born on March 14th, 1879.

Representing irrational numbers on number line:

Example: Represent each of the numbers $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$ on the real lin

Solution: Let X'OX be a horizontal line, taken as the X-axis and let O be the origin.





Let O represents 0.

i) Take OA = 1 unit and draw AB $\perp$ OA such that AB = 1 unit.

Join OB. Then, $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ units.

Write O as centre and OB as radius, drawn on arc, meeting OX at P.

Then, OP = OB = $\sqrt{2}$ units

Thus, the point P represents $\sqrt{2}$ on the real line.

ii) Now, draw BC $\perp$ OB such that BC = 1 unit.

Join OC. Then, $OC = \sqrt{OB^2 + BC^2} = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$ units

With O as centre and OC as radius, draw an arc, meeting OX at Q, then OQ = OC = $\sqrt{3}$ units.

Thus, the point Q represents $\sqrt{3}$ on the real line.

iii) Now, draw CD $\perp$ OC such that CD = 1 unit.

Join OD. Then, $OD = \sqrt{OC^2 + CD^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$ units

Now, draw DE $\perp$ OD such that DE = 1 unit

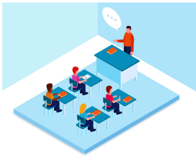
Join OE. Then, $OE = \sqrt{OD^2 + DE^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$ units

With O as centre and OE as radius draw an arc, meeting OX at R.

Then, OR = OE = $\sqrt{5}$ units.

Thus, the point R represents $\sqrt{5}$ on the real line.

Hence, the points P, Q, R represent the numbers $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$ respectively.



CLASSROOM DISCUSSION QUESTIONS CDQ 1.1

1. Which of the following is an irrational number?

(A) 0.33333333...

(B) 0.12112111211112...

(C) 0.5

(D) 0.75

2. According to the definition, which type of decimal numbers are considered irrational?

(A) Terminating decimals

(B) Repeating decimals

(C) Non-terminating, repeating decimals

(D) Non-terminating, non-repeating decimals

3. Who proved that $\sqrt{2}$ is an irrational number?

(A) Pythagoras

(B) Theodorus of Cyrene

(C) Archimedes

(D) Euclid

4. Which of the following numbers is irrational?

(A) 0.123123123...

(B) 0.88888888...

(C) 0.45454545...

(D) 0.66666666...

5. What is the value of 'e', Euler's number, rounded to two decimal places?

(A) 2.72 (B) 2.71

(C) 3.14 (D) 2.72

6. Which of the following statements about π is true?

(A) π can be expressed as a fraction.

(B) π is a terminating decimal.

(C) π is a repeating decimal.

(D) π is a non-terminating, non-repeating decimal.

7. According to the given example, which point on the real line represents the number 2?

(A) Point P

(B) Point Q

(C) Point R

(D) Point S

8. How is the number 5 represented on the real line according to the given example?

(A) Point P (B) Point Q

(C) Point R (D) Point S

9. In the example provided, what length does OC represent on the real line?

(A) 1 unit (B) 2 units

(C) 3 units (D) 4 units

10. Which mathematician showed that numbers like 3, 5, 6, 10, 11, 12, 13, 14, 15, and 17 are irrational?

(A) Pythagoras

(B) Euclid

(C) Theodorus of Cyrene

(D) Archimedes

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.2

Property: If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

Example: Find two irrational numbers between 2 and 3.

Solution: An irrational number between 2 and 3 is $\sqrt{2\times3}=\sqrt{6}$.

Similarly an irrational number between 2 and $\sqrt{6}$ = $\sqrt{2\times\sqrt{6}}=\sqrt{2\sqrt{6}}$

$\therefore$ Required numbers are $\sqrt{6}$ and $\sqrt{2\sqrt{6}}$ lie between 2 and 3.

Example: Find three irrationals between 3 and 4.

Solution: An irrational number between 3 and 4 is $\sqrt{12}$

An irrational number between 3 and $\sqrt{12}$ is $\sqrt{3\sqrt{12}}$ = $\sqrt{6\sqrt{3}}$

Another irrational number between $\sqrt{12}$ and 4 is $\sqrt{4\sqrt{12}}$ = $\sqrt{8\sqrt{3}}$

$\therefore$ Required numbers are $\sqrt{6\sqrt{3}}$, $\sqrt{12}$ and $\sqrt{8\sqrt{3}}$ lie between 3 and 4.

Properties of Irrational Numbers:

1. Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.
2. i) Sum of two irrationals need not be an irrational.

   Example: Each one of $(2+\sqrt{3})$ and $(4-\sqrt{3})$ is irrational.

   But, $(2+\sqrt{3})+(4-\sqrt{3})=6$, which is rational.

   ii) Difference of two irrationals need not be an irrational.

   Example: Each one of $(5+\sqrt{2})$ and $(3+\sqrt{2})$ is irrational.

   But $(5+\sqrt{2})-(3+\sqrt{2})=2$, which is rational.

   iii) Product of two irrationals need not be an irrational.

   Example: $\sqrt{3}$ is irrational. But $\sqrt{3}\times\sqrt{3}=3$, which is rational.

   iv) Quotient of two irrationals need not be an irrational.

   Example: Each one of $2\sqrt{3}$ and $\sqrt{3}$ is irrational. But $\frac{2\sqrt{3}}{\sqrt{3}}=2$, which is rational.

SURDS: If 'n' is a positive integer and a rational number a (> 0) is not the n^{th} power of any other rational number, then $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ is called a 'surd' or 'radical' of order n and it can be read as n^{th} root of a.

Irrational Numbers and Surds

Let $a \in Q^+$, $n \in I^+$ such that $\sqrt[n]{a}$ is an irrational then $\sqrt[n]{a}$ is called a surd.

Here, $a^{\frac{1}{n}}$ is called an exponential form of a surd and $\sqrt[n]{a}$ is called radical form of a surd.

The square roots of numbers that do not have exact square roots are called Surds.

Note: In a surd $\sqrt[n]{a}$,

i) The symbol $\sqrt[n]{\ }$ is called radical sign.

ii) 'a' is called the radicand.

iii) 'n' is called the order of the surd.

Example: In a surd $\sqrt[3]{5}$, 5 is the radicand, 3 is the order of the surd and $\sqrt[3]{\ }$ is radical sign.

SURDS (RADICALS)	IRRATIONAL NUMBERS
If $n \in I^+$, $a(>0) \in Q$ and 'a' is not n^{th} power of any other rational number then $\sqrt[n]{a}$ is called a surd.	Number which is neither terminating nor repeating decimal.
Examples for surds are $\sqrt{2}, \sqrt{8}, \sqrt[7]{5}, \ldots$	Examples for irrationals are $\sqrt{2}, \sqrt{8}, \sqrt[7]{5}, \pi, e, \ldots$
Every surd is an irrational.	Every irrational number is need not be a surd π, e are not surds.

Note: i) $\sqrt{3}, \sqrt[3]{2}, \sqrt[4]{11}, \frac{2}{3}\sqrt[3]{10}$, are surds.

ii) In general, $\sqrt[2]{a}$ is written as $\sqrt{a}$.

iii) The exponential form of $\sqrt{5}$ is $5^{1/2}$ and $\sqrt{5}$ is called the radical form.

iv) 0.5454454. . . is not a surd

v) $\sqrt{2}$ is a surd and also irrational, but π is only irrational and not a surd.

vi) $\sqrt{\pi}, \sqrt[3]{e}$ are irrationals but not surds.

Types of surds based on Order:

Quadratic surd: A surd of order two is called a quadratic surd.

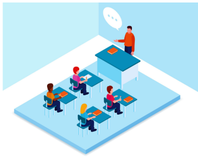
Example: $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc.

Cubic surd: A surd of order three is called a cubic surd.

Example: $\sqrt[3]{4}, \sqrt[3]{25}, \sqrt[3]{30}$ etc.

Biquadratic surd: A surd of order four is called a biquadratic surd.

Example: $\sqrt[4]{2}, \sqrt[4]{8}, \sqrt[4]{10}$



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.2

1. What property states that if a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then ab is an irrational number lying between a and b?

(A) Associative property
(B) Distributive property
(C) Commutative property
(D) Property of irrationals

2. How many irrational numbers between 2 and 3 were found in the given example?

(A) One (B) Two
(C) Three (D) None

3. According to the given example, what is the irrational number found between 2 and 6?

(A) 12 (B) 6
(C) 26 (D) 18

4. Which property of irrational numbers states that the sum of two irrationals need not be an irrational?

(A) Property 1 (B) Property 2i
(C) Property 2ii (D) Property 2iii

5. In the example provided, what operation of two irrationals resulted in a rational number?

(A) Addition
(B) Subtraction
(C) Multiplication
(D) Division

6. Which type of surd is a surd of order two?

(A) Quadratic surd
(B) Cubic surd
(C) Biquadratic surd
(D) Linear surd

7. What is the order of the surd $\sqrt[3]{4}$?

(A) 2 (B) 3
(C) 4 (D) 5

8. In the surd $\sqrt[3]{25}$, what is the radicand?

(A) 3 (B) 4
(C) 5 (D) 25

9. Which of the following numbers is an example of a cubic surd?

(A) $\sqrt[3]{4}$ (B) $\sqrt[3]{25}$
(C) $\sqrt[3]{30}$ (D) $\sqrt[3]{8}$

10. What is another name for the surd $\sqrt[4]{2}$?

(A) Quadratic surd
(B) Cubic surd
(C) Biquadratic surd
(D) Quartic surd

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6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.3

Types of surds based on Terms:

Simple surd: A surd which consists of a single term is called a simple or monomial surd.

Example: $\sqrt[2]{3}, \sqrt[3]{4}, \sqrt[n]{k}$, etc.

Compound surd: The sum or difference of a rational number and one or more surds is called a compound surd if the resultant is also a surd.

Example: $3+\sqrt{3},\ 5+\sqrt{3}-\sqrt{7},\ \sqrt{5}+\sqrt{7}-3\sqrt{4}$, etc.

Note: $\pi+\sqrt{2}-\sqrt[3]{5}$ is an irrational number but not a compound surd.

Since $\pi+\sqrt{2}-\sqrt[3]{5}$ is not a surd.

Binomial surd: A compound surd consisting of two terms is called a binomial surd.

Example: $\sqrt{2}+\sqrt{5},\ 6-\sqrt{7},\ \sqrt{3}-5$, etc.

Trinomial surd: A compound surd consisting of three terms is called a trinomial surd.

Example: $6-\sqrt{3}+\sqrt{5},\ 5+\sqrt{10}+\sqrt[3]{20},\ \sqrt{3}+\sqrt{2}-\sqrt{7}$, etc.

Similar surds: If two surds are of different multiples of the same simple surd, then they are called the similar surds or like surds other wise, they are called dissimilar surds or unlike surds.

Example: **i)** $2\sqrt{5}, 3\sqrt{5}, 5\sqrt{5}$ are similar surds or like surds.

ii) $2\sqrt{3}, 2\sqrt{5}, 7\sqrt{2}$ are dissimilar surds or unlike surds.

Note: **i)** The product of two similar quadratic surds is a rational number.

Example: $(3\sqrt{3})(5\sqrt{3}) = (3\times5)(\sqrt{3}\times\sqrt{3}) = 15\times3 = 45 \in Q$

ii) The quotient of two similar surds is a rational number

Example: $7\sqrt[3]{4} \div 2\sqrt[3]{4} = \frac{7}{2} \in Q$

Pure surd: A surd expressed in the form $a\sqrt[n]{b}$, where a = 1, is called a pure surd or an entire surd.

Example: $\sqrt{6}, \sqrt{20}, \sqrt[3]{5}, \sqrt[3]{25}$ etc.

Mixed surd: If 'a' is a non-zero rational number and $\sqrt[n]{b}$ is a monomial surd, then $a \pm \sqrt[n]{b}, a\sqrt[n]{b}$ are called mixed surds.

Example: $2\sqrt{3}, 4\sqrt[7]{9}, 9-\sqrt{2}, 7+\sqrt[3]{5}$ etc.

Equal surds: Let a, b, c and d are rational numbers. If two surds $a+\sqrt{b} = c+\sqrt{d}$, then a = c and b = d.

Example: If $x+\sqrt{5} = 3+\sqrt{y}$, then x = 3 and y = 5.

Simplest form of a surd: A surd, expressed in the form $a\sqrt[n]{b}$, where 'b' is the least positive rational number, is called the simplest form of the given surd.

Example: **i)** The entire form of $2\sqrt{10}$ is $\sqrt{10\times2^2} = \sqrt{10\times4} = \sqrt{40}$

ii) The simplest form of $\sqrt{32}$ is $\sqrt{2\times16} = 4\sqrt{2}$

Laws of radicals: The radicals $\sqrt[n]{a}$, $\sqrt[n]{b}$ are the positive n^{th} root of positive rational numbers a, b respectively for any positive integers m, n, p such that

i) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a^{\frac{n}{n}} = a$

ii) $\sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}$

iii) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

iv) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

v) $\sqrt[n]{a^p} = \sqrt[n]{\sqrt[m]{(a^p)^m}} = \sqrt[mn]{a^{pm}}$

CLASSROOM DISCUSSION QUESTIONS — CDQ 1.3

1. What type of surd consists of a single term?

(A) Compound surd

(B) Binomial surd

(C) Trinomial surd

(D) Simple surd

2. Which of the following is an example of a compound surd?

(A) 3+4 (B) 5–2

(C) 2+5–3 (D) $3+\pi-3$

3. What type of surd consists of two terms?

(A) Compound surd

(B) Trinomial surd

(C) Binomial surd

(D) Simple surd

4. If two surds are different multiples of the same simple surd, what are they called?

(A) Similar surds

(B) Dissimilar surds

(C) Mixed surds

(D) Pure surds

5. Which product results in a rational number according to the given properties?

(A) $(3\sqrt{3})(5\sqrt{3})$ (B) $(2\sqrt{2})(3\sqrt{3})$

(C) $(7\sqrt{3})(4\sqrt{2})$ (D) $(3\sqrt{2})(4\sqrt{2})$

6. What type of surd is expressed in the form $a+\sqrt{b}$, where a is a rational number and $\sqrt{b}$ is a simple surd?

(A) Mixed surd

(B) Pure surd

(C) Compound surd

(D) Simplest form surd

7. If a+b=c+d, what can be concluded according to the properties of equal surds?

(A) a=c and b=d

(B) a=b=c=d

(C) a=c or b=d

(D) $a\neq c$ and $b\neq d$

8. What is the simplest form of the surd $2\sqrt{10}$?

(A) 10 (B) 20

(C) 40 (D) $5\sqrt{2}$

9. According to the laws of radicals, what does $n\sqrt{a}\times n\sqrt{b}$ equal?

(A) $n\sqrt{ab}$ (B) $n^2\sqrt{ab}$

(C) $n\sqrt{a}\sqrt{b}$ (D) $n\sqrt{a+b}$

10. Which law of radicals states that $m\sqrt[n]{a}$ is equal to $mn\sqrt{a}$?

(A) Law ii

(B) Law iii

(C) Law iv

(D) Law v

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CONCEPT 1.4

Uses of laws of radicals: By using the laws of radicals, we can

i) Convert a pure surd into a mixed surd

ii) Convert a mixed surd into pure surd

iii) Reduce the given surds to the same order

iv) Compare the given monomial surds.

Convert a pure surd in to a mixed surd:

Example: Express $\sqrt[5]{288}$ as a mixed surd in its simplest form.

Solution: $\sqrt[5]{288} = \sqrt[5]{2^5 \times 3^2} = \sqrt[5]{2^5} \times \sqrt[5]{9} = 2\sqrt[5]{9}$

Example: Express $\sqrt[3]{72}$ as a mixed surd in its simplest form.

Solution: $\sqrt[3]{72} = \sqrt[3]{2^3 \times 3^2} = \sqrt[3]{2^3} \times \sqrt[3]{3^2} = 2\sqrt[3]{9}$

Example: Convert $\sqrt{\frac{50}{4}}$ into its simplest form.

Solution: $\sqrt{\frac{50}{4}} = \sqrt{\frac{25 \times 2}{4}} = \sqrt{\frac{5^2 \times 2}{2^2}} = \sqrt{\left(\frac{5}{2}\right)^2 \times 2} = \frac{5}{2}\sqrt{2}$

Example: Write the simplest form of $\sqrt[3]{24}$

Solution: $\sqrt[3]{24} = \sqrt[3]{8 \times 3} = \sqrt[3]{2^3 \times 3} = 2\sqrt[3]{3}$

Convert a mixed surd in to pure surd:

Example: Convert $\frac{2}{3}\sqrt{5}$ into pure surd.

Solution: $\frac{2}{3}\sqrt{5} = \sqrt{\left(\frac{2}{3}\right)^2 \times 5} = \sqrt{\frac{4}{9} \times 5} = \sqrt{\frac{20}{9}}$

Example: Express $5\sqrt{6}$ as a pure surd.

Solution: $5\sqrt{6} = \sqrt{5^2 \times 6} = \sqrt{150}$

Example: Express $3\sqrt[4]{5}$ as a pure surd.

Solution: $3\sqrt[4]{5} = \sqrt[4]{3^4 \times 5} = \sqrt[4]{405}$

Reduce the given surds to the same form or order:

Example: Convert the surds $\sqrt[3]{3}$, $\sqrt[4]{4}$ into the same order.

Solution: We can write the given two surds into the exponent form as $3^{1/3}$ and $4^{1/4}$.

The exponents $\frac{1}{3}$ and $\frac{1}{4}$ are to be written such that they have a common denominator.

We know that L.C.M of 3 and 4 is 12.

So we can write $\frac{1}{3}$ as $\frac{1}{3}\times\frac{4}{4} = \frac{4}{12}$ and $\frac{1}{4}$ as $\frac{1}{4}\times\frac{3}{3} = \frac{3}{12}$

$\therefore\ 3^{1/3} = 3^{4/12} = (3^4)^{1/12} = \sqrt[12]{81}$ and

$4^{1/4} = 4^{3/12} = (4^3)^{1/12} = \sqrt[12]{64}$

$\therefore\ \sqrt[3]{3}$ and $\sqrt[4]{4}$ can be written as $\sqrt[12]{81}$ and $\sqrt[12]{64}$

Example: Convert the surds $\sqrt[4]{10}$, $\sqrt[3]{6}$ and $\sqrt{3}$ into the same order.

Solution: $\sqrt[4]{10} = 10^{\frac{1}{4}}$, $\sqrt[3]{6} = 6^{\frac{1}{3}}$ and $\sqrt{3} = 3^{\frac{1}{2}}$

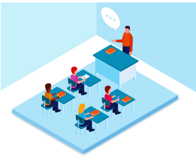
Now the L.C.M of 4, 3 and 2 is 12.

$\therefore\ \sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{1}{4}\times\frac{3}{3}} = \sqrt[12]{10^3} = \sqrt[12]{1000}$

$\sqrt[3]{6} = 6^{\frac{1}{3}} = 6^{\frac{1}{3}\times\frac{4}{4}} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{1}{2}\times\frac{6}{6}} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\therefore\ \sqrt[4]{10}$, $\sqrt[3]{6}$ and $\sqrt{3}$ can be written as $\sqrt[12]{1000}$, $\sqrt[12]{1296}$ and $\sqrt[12]{729}$.



CLASSROOM DISCUSSION QUESTIONS CDQ 1.4

1. Which of the following is not a use of the laws of radicals?

(A) Convert a pure surd into a mixed surd
(B) Convert a mixed surd into a pure surd
(C) Reduce given surds to the same order
(D) Simplify rational numbers

2. How can $5\sqrt{288}$ be expressed as a mixed surd in its simplest form?

(A) $5\sqrt{25}\times\sqrt{32}$ (B) $5\sqrt{25}\times\sqrt{9}$
(C) $5\sqrt{25}\times\sqrt{81}$ (D) $5\sqrt{81}\times\sqrt{9}$

3. What is the simplest form of $3\sqrt{72}$ expressed as a mixed surd?

(A) $3\sqrt{23}\times\sqrt{32}$ (B) $3\sqrt{23}\times\sqrt{3}$
(C) $3\sqrt{32}\times\sqrt{9}$ (D) $5\sqrt{81}\times\sqrt{9}$

4. How can $\frac{50}{4}$ be simplified into its simplest form?

(A) $\frac{25}{2}\times\frac{2}{2}\times\frac{5}{2}$ (B) $\frac{25}{4}\times\frac{2}{2}\times\frac{5}{2}$
(C) $\frac{50}{2}\times\frac{2}{2}\times\frac{5}{2}$ (D) $\frac{50}{4}\times\frac{2}{2}\times\frac{5}{2}$

5. What is the simplest form of $3\sqrt{24}$

(A) $3\sqrt{23}\times\sqrt{3}$ (B) $3\sqrt{8}\times\sqrt{3}$
(C) $3\sqrt{23}\times\sqrt{8}$ (D) $3\sqrt{8}\times\sqrt{3}$

6. How can $2\sqrt{5}$ be converted into a pure surd?

(A) $2\times\sqrt{5^2}$ (B) $2\times\sqrt{10}$
(C) $5^2\times2$ (D) $2\times\sqrt{10^2}$

7. What is the pure surd form of $5\sqrt{6}$

(A) $5^2\times6$ (B) $5\times\sqrt{6^2}$
(C) 6×5^2 (D) $5^2\times\sqrt{6}$

8. How can the surds $3\sqrt{3}$ and $4\sqrt{4}$ be converted to the same order?

(A) Express both as $3^{1/3}$ and $4^{1/4}$
(B) Express both as $3^{1/4}$ and $4^{1/3}$
(C) Express both as $3^{1/3}$ and $4^{1/3}$
(D) Express both as $3^{1/4}$ and $4^{1/4}$

9. Which of the following sets of surds are expressed in the same order?

(A) $10\sqrt{4}, 6\sqrt{3}, \sqrt{3}$
(B) $10\sqrt{4}, 8\sqrt{6}, 5$
(C) $10\sqrt{4}, 8\sqrt{6}, \sqrt{3}$
(D) $10\sqrt{4}, 6\sqrt{3}, 3\sqrt{2}$

10. How can the surds $4\sqrt{10}$ and $3\sqrt{6}$ be converted to the same order?

(A) Express both as $10^{1/2}$ and $6^{1/2}$
(B) Express both as $10^{1/3}$ and $6^{1/3}$
(C) Express both as $10^{1/2}$ and $6^{1/3}$
(D) Express both as $10^{1/3}$ and $6^{1/2}$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.5

Compare the given monomial surds: Comparison of surds is possible only when they are of the same order. The radicands are then to be compared.

Thus, $\sqrt[3]{5}$ and $\sqrt[3]{7}$ can be compared.

Since the order of the surds are same, now we can compare radicands.

Since 7 > 5 $\Rightarrow$ $\sqrt[3]{7} > \sqrt[3]{5}$.

$\therefore$ $\sqrt[3]{7}$ is greater than $\sqrt[3]{5}$.

Note: In order to compare the surds of different order and different base we first reduce them into same order.

Example: Which is greater $\sqrt[3]{3}$ or $\sqrt[4]{5}$

Solution: The order of the surds are different. So convert them into the same order.

$$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{1}{3}\times\frac{4}{4}} = \sqrt[12]{3^4} = \sqrt[12]{81}, \quad \sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{1}{4}\times\frac{3}{3}} = \sqrt[12]{5^3} = \sqrt[12]{125}$$

Since 125 > 81

$\Rightarrow$ $\sqrt[12]{125} > \sqrt[12]{81}$

$\Rightarrow$ $\sqrt[4]{5} > \sqrt[3]{3}$

$\therefore$ $\sqrt[4]{5}$ is greater than $\sqrt[3]{3}$

Example: Arrange the surds in an ascending order of their magnitudes $\sqrt[3]{2}, \sqrt[9]{4}, \sqrt[6]{3}$.

Solution: The L.C.M of, 3, 9 and 6 is 18.

$$\sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\frac{1}{3}\times\frac{6}{6}} = \sqrt[18]{2^6} = \sqrt[18]{64}$$

$$\sqrt[9]{4} = 4^{\frac{1}{9}} = 4^{\frac{1}{9}\times\frac{2}{2}} = \sqrt[18]{4^2} = \sqrt[18]{16}$$

$$\sqrt[6]{3} = 3^{\frac{1}{6}} = 3^{\frac{1}{6}\times\frac{3}{3}} = \sqrt[18]{3^3} = \sqrt[18]{27}$$

Since 16 < 27 < 64 $\Rightarrow \sqrt[18]{16} < \sqrt[18]{27} < \sqrt[18]{64} \Rightarrow \sqrt[9]{4} < \sqrt[6]{3} < \sqrt[3]{2}$

$\therefore$ The ascending order is $\sqrt[9]{4}$, $\sqrt[6]{3}$ and $\sqrt[3]{2}$.

Addition and Subtraction of Surds:

Two surds can be added or subtracted one from the other by using distributive law, only when they are similar surds. We cannot add or subtract dissimilar surds.

i.e., $a\sqrt{c}+b\sqrt{c}=(a+b)\sqrt{c}$ ($\because$ distributive law) and $a\sqrt{c}-b\sqrt{c}=(a-b)\sqrt{c}$

Example: i) $2\sqrt{6}+4\sqrt{6}=(2+4)\sqrt{6}=6\sqrt{6}$

ii) $4\sqrt{2}+3\sqrt{2}-10\sqrt{2}=(4+3-10)\sqrt{2}=-3\sqrt{2}$

Example: Simplify: $8\sqrt{3}-4\sqrt{75}+3\sqrt{300}$

Solution: $8\sqrt{3}-4\sqrt{3\times25}+3\sqrt{3\times100}$

$=8\sqrt{3}-4\sqrt{3\times5^2}+3\sqrt{3\times10^2}=8\sqrt{3}-(4\times5)\sqrt{3}+(3\times10)\sqrt{3}$

$=8\sqrt{3}-20\sqrt{3}+30\sqrt{3}=(8-20+30)\sqrt{3}=18\sqrt{3}$

Multiplication and Division of Surds:

If $\sqrt[n]{a}, \sqrt[n]{b}$ are two surds of the same order, then their multiplication can be defined as $\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{a\times b}$ and their division can be defined as $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$, where $b\neq0$.

Note: When the surds to be multiplied or divided are not of the same order, they have to be brought to the same order by using the laws of radicals before the operation is done.

Example: $\sqrt[3]{48}\div\sqrt[3]{162}=\frac{\sqrt[3]{48}}{\sqrt[3]{162}}=\frac{\sqrt[3]{8\times6}}{\sqrt[3]{27\times6}}=\frac{2\sqrt[3]{6}}{3\sqrt[3]{6}}=\frac{2}{3}$.

Example: Simplify: $\frac{\sqrt[3]{2}\times\sqrt[6]{6}}{\sqrt[6]{3}\times\sqrt[6]{8}}$

Solution: Numerator $=\sqrt[3]{2}\times\sqrt[6]{6}=\sqrt[(3\times2)]{2^2}\times\sqrt[6]{6}\left[\because \sqrt[m]{p}=\sqrt[m\times n]{p^n}\right]$

$=\sqrt[6]{4}\times\sqrt[6]{6}=\sqrt[6]{24}\ \left(\because \sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{a\times b}\right)$

Denominator $=\sqrt[6]{3}\times\sqrt[6]{8}=\sqrt[6]{3\times8}=\sqrt[6]{24}$

$\therefore \frac{\sqrt[3]{2}\times\sqrt[6]{6}}{\sqrt[6]{3}\times\sqrt[6]{8}}=\frac{\sqrt[6]{24}}{\sqrt[6]{24}}=1$

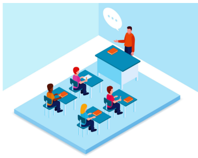
Example: Multiply: $\sqrt[3]{2}$ by $\sqrt[5]{3}$.

Solution: $\sqrt[3]{2}=2^{\frac{1}{3}};\ \sqrt[5]{3}=3^{\frac{1}{5}}$

L.C.M of 3, 5 is 15.

So $\sqrt[3]{2}=2^{\frac{1}{3}}=2^{\frac{5}{15}};\ \sqrt[5]{3}=3^{\frac{1}{5}}=3^{\frac{3}{15}}$

Hence, $\sqrt[3]{2}\times\sqrt[5]{3}=2^{\frac{5}{15}}\times3^{\frac{3}{15}}=(2^5)^{\frac{1}{15}}\times(3^3)^{\frac{1}{15}}=\sqrt[15]{2^5\times3^3}=\sqrt[15]{32\times27}=\sqrt[15]{864}$.



CLASSROOM DISCUSSION QUESTIONS CDQ 1.5

1. Which surd is greater: $3\sqrt{5}$ or $3\sqrt{7}$?

(A) $3\sqrt{5}$ (B) $3\sqrt{7}$
(C) They are equal
(D) Comparison cannot be made

2. Arrange the following surds in ascending order: $3\sqrt{2}, 9\sqrt{4}, 6\sqrt{3}$

(A) $3\sqrt{2}, 9\sqrt{4}, 6\sqrt{3}$
(B) $9\sqrt{4}, 3\sqrt{2}, 6\sqrt{3}$
(C) $3\sqrt{2}, 6\sqrt{3}, 9\sqrt{4}$
(D) $6\sqrt{3}, 3\sqrt{2}, 9\sqrt{4}$

3. What is the result of $8\sqrt{3} - 4\sqrt{75} + 3\sqrt{300}$?

(A) $11\sqrt{3}$ (B) $18\sqrt{3}$
(C) $15\sqrt{3}$ (D) $9\sqrt{3}$

4. How can $3\sqrt{6} \div 6\sqrt{6}$ be simplified?

(A) $\sqrt{6}$ (B) 1
(C) $\frac{1}{6}$ (D) $\frac{\sqrt{6}}{6}$

5. What is the product of $3\sqrt{2}$ and $5\sqrt{3}$?

(A) $15\sqrt{6}$ (B) $15\sqrt{5}$
(C) $3\sqrt{6} + 5\sqrt{3}$ (D) $8\sqrt{6}$

6. Which law is used to simplify $3\sqrt{2} \times 5\sqrt{3}$?

(A) Distributive law
(B) Associative law
(C) Commutative law
(D) Identity law

7. What is the result of $\sqrt{32} \times \sqrt{72}$?

(A) $\sqrt{16}$ (B) $\sqrt{8}$
(C) 8 (D) 4

8. What is the division of $3\sqrt{6}$ by $6\sqrt{6}$?

(A) $\frac{\sqrt{6}}{6}$ (B) 1
(C) $\frac{1}{6}$ (D) $\sqrt{6}$

9. Which law is used to simplify $3\sqrt{6} \div 6\sqrt{6}$?

(A) Distributive law
(B) Commutative law
(C) Associative law
(D) Identity law

10. How can $3\sqrt{2}$ and $5\sqrt{3}$ be multiplied to get the result $15\sqrt{6}$?

(A) They cannot be multiplied to get $15\sqrt{6}$
(B) By using the distributive law
(C) By using the commutative law
(D) By using the associative law

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.6

Rationalisation:

If the product of two surds is a rational number, then each of them is called a rationalizing factor (R.F) of the other.

Example: $\sqrt[5]{3}\times\sqrt[5]{81}=\sqrt[5]{3}\times\sqrt[5]{3^4}=\sqrt[5]{3^5}=3$ is a rational number.

$\therefore$ The R.F of $\sqrt[5]{3}$ is $\sqrt[5]{81}$ and the R.F of $\sqrt[5]{81}$ is $\sqrt[5]{3}$

Example: $\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)=\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2=7-3=4\in Q$

$\sqrt{7}+\sqrt{3}$ is a R.F of $\sqrt{7}-\sqrt{3}$ and $\sqrt{7}-\sqrt{3}$ is a R.F. of $\sqrt{7}+\sqrt{3}$

Note: The R.F. of a given surd is not unique. A surd has infinite number of rationalizing factors.

Rationalisation of monomial surds:

Example: The R.F of $\sqrt{3}$ are $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \sqrt{3}, 2\sqrt{3}, 3\sqrt{3}$,...... But the simplest R.F is $\sqrt{3}$.

Note: The simplest R.F of $\sqrt[n]{a^m}$ is $\sqrt[n]{a^{n-m}}$, where a > 0 and m, $n\in z^+$

Example: $\sqrt{3}, 3\sqrt{3}$ are R.F of $6\sqrt{3}$.

Because $\sqrt{3}\times6\sqrt{3}=6\times(\sqrt{3}\times\sqrt{3})=6\times3=18\in Q$

$3\sqrt{3}\times6\sqrt{3}=(3\times6)\times(\sqrt{3}\times\sqrt{3})=18\times3=54\in Q$

Example: $2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots$ are R.F.'s of $5\sqrt{2}$.

If one R.F. of a surd is known, then the product of its factor by a non-zero rational number is also a R.F. of the given surd.

Example: **i)** If R.F of $\sqrt{50}$ is $\sqrt{2}$, then $2\sqrt{2}, 3\sqrt{2}, -\frac{2}{3}\sqrt{2}$ etc. are R.F's of $\sqrt{50}$.

Here, $\sqrt{2}$ is called the simplest R.F of $\sqrt{50}$ $\left(\because \sqrt{50}=5\sqrt{2}\right)$

ii) The simplest R.F. of $\sqrt{125}$ is $\sqrt{5}$

iii) The simplest R.F. of $\sqrt[3]{36}$ is $\sqrt[3]{6}$

Conjugate Surds: If the sum and product of two surds is a rational number, then they are said to be conjugate to each other.

Example: $2+\sqrt{3},\ 2-\sqrt{3}$ are conjugate to each other.

Solution: Sum $= 2+\sqrt{3}+2-\sqrt{3} = 4 \in Q$

Product $= \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) = 4-3 = 1 \in Q$

Here the sum and product of the given two surds is a rational number.

Hence they are conjugate to each other.

Note: i) Every conjugate surd is a rationalizing factor but converse need not be true.

ii) If the conjugate of a surd exists, then it is unique, where as R.F's of it are infinite.

iii) The conjugate exists only when the surds are of the form $a+\sqrt{b}$ and $a-\sqrt{b}$.

iv) The simplest rationalizing factor of a binomial quadratic surd is its conjugate surd.

Example: Find the conjugate of $\sqrt{2}+\sqrt{3}$

Solution: The conjugate of $\sqrt{2}+\sqrt{3}$ does not exist.

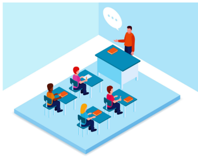
Since $\left(\sqrt{2}+\sqrt{3}\right)+\left(\sqrt{2}-\sqrt{3}\right) = 2\sqrt{2}$ is not a rational.

$\therefore \sqrt{2}-\sqrt{3}$ is a R.F of $\sqrt{2}+\sqrt{3}$ but not a conjugate.

Example: The conjugate of $\sqrt{3}-4$ is $-\sqrt{3}-4$

Check: Sum $= \left(-4+\sqrt{3}\right)+\left(-4-\sqrt{3}\right) = -8$, is a rational number

Product $= \left(-4+\sqrt{3}\right)\left(-4-\sqrt{3}\right) = 16-3 = 13$, is a rational number.



CLASSROOM DISCUSSION QUESTIONS CDQ 1.6

1. Which of the following is a rationalizing factor of ?

(A) $3\sqrt{5}$ (B) $5\sqrt{3}$

(C) $3\sqrt{15}$ (D) $15\sqrt{3}$

2. What is the simplest rationalizing factor of $6\sqrt{3}$?

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$

(C) $3\sqrt{3}$ (D) $6\sqrt{3}$

3. If the conjugate of 2+3 does not exist, what is its rationalizing factor?

(A) 3–2 (B) 3+2

(C) 3–3 (D) 3+3

4. Which pair of surds are conjugate to each other?

(A) 2+3 and 2–3

(B) $2\sqrt{3}$ and $3\sqrt{2}$

(C) $3\sqrt{5}$ and $5\sqrt{3}$

(D) $4\sqrt{2}$ and $2\sqrt{4}$

5. If 50 has a rationalizing factor of 2, what are some other rationalizing factors?

(A) 4, 6, 8

(B) 3, 5, 7

(C) $2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

(D) $2\sqrt{2}, 3\sqrt{3}, 4\sqrt{3}$

6. What is the product of $3\sqrt{2}$ and its simplest rationalizing factor?

(A) $6\sqrt{2}$ (B) $3\sqrt{4}$

(C) 6 (D) $3\sqrt{2}$

7. Which statement about conjugate surds is NOT true?

(A) Every conjugate surd is a rationalizing factor

(B) The conjugate surd exists only when the surds are of the form a+b and a–b

(C) The product of conjugate surds is always irrational

(D) If the conjugate of a surd exists, it is unique

8. What is the conjugate of 3–4?

(A) 3–4 (B) –3–4

(C) 3+4 (D) –3+4

9. If the sum of two surds is rational, what can be concluded about them?

(A) They are conjugate surds

(B) They have the same rationalizing factor

(C) They are irrational

(D) They are not surds

10. Which of the following is NOT a property of rationalizing factors?

(A) Every surd has a unique rationalizing factor

(B) A surd has infinite rationalizing factors

(C) The product of two rationalizing factors is rational

(D) Rationalizing factors are used to convert surds into rational numbers

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

C.D.F.

CONCEPTS, DEFINITIONS AND FORMULAE

1. If a, b $\in Q^+$ such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.
2. Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.
3. Product of two irrationals need not be an irrational.
4. Let $a \in Q, n \in I^+$ such that $\sqrt[n]{a}$ is an irrational then $\sqrt[n]{a}$ is called a surd.
5. In a surd $\sqrt[n]{a}$, 'n' is called its order, 'a' is called the radicand, symbol $\sqrt[n]{\ }$ is radical sign.
6. Every surd is an irrational number, but every irrational is need not be a surd.
7. The product of two similar quadratic surds is a rational number.
8. A surd expressed in the form $a\sqrt[n]{b}$, where a = 1, is called a pure surd or an entire surd.
9. If $a+\sqrt{b}=c+\sqrt{d}$, then a = c and b = d.
10. In order to compare the surds of different order and different base we first reduce them into same order.
11. $\sqrt{c}$ is any surd and a, b are rational numbers such that $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$.
12. If $\sqrt[n]{a}, \sqrt[n]{b}$ are two surds of the same order, then their multiplication can be defined as $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$ and their division can be defined as $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, where $b \neq 0$.
13. If the product of two surds is a rational number, then each of them is called a rationalizing factor (R.F) of the other.
14. A surd has infinite number of rationalizing factors.
15. The simplest R.F of $\sqrt[n]{a^m}$ is $\sqrt[n]{a^{n-m}}$, where a > 0 and m, n $\in z^+$.
16. If the sum and product of two surds is a rational number, then they are said to be conjugate to each other.
17. Every conjugate surds is a rationalizing factor but converse need not be true.
18. The conjugate exists only when the surds are of the form $a \pm \sqrt{b}$.

ADVANCED WORKSHEET

Single Correct Answer Type (S.C.A.T)

1. A number which can neither be expressed as a terminating decimal nor as repeating decimal is called:

(A) Rational number

(B) Irrational number

(C) Both (A) and (B)

(D) None

2. Numbers which cannot be expressed in the form $\frac{p}{q}$, $q \neq 0$, p, $q \in Z$ are called _______.

(A) rational

(B) irrational

(C) fractional

(D) decimal

3. $\sqrt{2}$ is an __________ number.

(A) rational

(B) Irrational

(C) natural

(D) fractional

4. Decimal representation of $\sqrt{2}$ to first four places is ________.

(A) 1.4145

(B) 1.4143

(C) 1.4142

(D) 1.4144

5. The value of $\sqrt{3}$ to first three decimal places is __________.

(A) 1.734

(B) 1.736

(C) 1.730

(D) 1.732

6. Given $\sqrt{3} = 1.732$, then the value of $\sqrt{12}$ is _________.

(A) 3.464

(B) 3.467

(C) 2.464

(D) 3.454

7. In a surd $\sqrt[n]{a}$, 'n' is called _____ of the surd.

(A) radicand

(B) radical

(C) radical sign

(D) order

8. Which of the following is rational number?

(A) $\left(3+\sqrt{5}\right)$

(B) $\left(-1+\sqrt{3}\right)$

(C) $5\sqrt{16}$

(D) $-\sqrt{7}$

9. Which of the following is an irrational number?

(A) $\sqrt{\frac{4}{9}}$

(B) $\frac{4}{5}$

(C) $\sqrt{81}$

(D) $\sqrt{7}$

10. Which of the following is an irrational number?

(A) $\sqrt{4}$

(B) $\sqrt{5}$

(C) 1.5

(D) $1.\bar{5}$

11. The sum and product of $a+\sqrt{b}$ and $a-\sqrt{b}$ are rational then the surds are ______ each other.

(A) Rationalizing

(B) Simple

(C) Conjugate

(D) Pure

12. Number of conjugate of a surd are _____.

(A) 0

(B) Infinite

(C) 1

(D) Does not exist

13. Multiply: $\sqrt{14}$ by $\sqrt{8}$

(A) $4\sqrt{7}$

(B) $2\sqrt{14}$

(C) 24

(D) $\sqrt{28}$

14. The number of irrationals in the given list $\sqrt{3}, \pi, \frac{1}{3}, 0, \sqrt[5]{2}, \frac{22}{7}, \sqrt{36}$ is

(A) 3

(B) 4

(C) 5

(D) 6

15. $10\sqrt{3}$ and $11\sqrt{3}$ are ______ surds.

(A) Pure

(B) Similar

(C) Binomial

(D) Dissimilar

16. Express the surd $\sqrt[4]{1024}$ in the simplest form as multiples of smaller surds.

(A) $2\sqrt[4]{2}$

(B) $4\sqrt{2}$

(C) $2\sqrt{4}$

(D) $4\sqrt[4]{4}$

17. The simplest form of $\sqrt[3]{108}$ is:

(A) $5\sqrt[3]{4}$

(B) $3\sqrt[3]{4}$

(C) $3\sqrt[3]{2}$

(D) $10\sqrt[3]{4}$

18. The simplest form of $5\sqrt{\frac{1}{10}}$ is:

(A) $\sqrt{\frac{5}{4}}$

(B) $\sqrt{\frac{2}{4}}$

(C) $\sqrt{\frac{6}{4}}$

(D) $\sqrt{\frac{5}{2}}$

19. The ascending order of $\sqrt[3]{9}, \sqrt[9]{5}, \sqrt[3]{7}$ is:

(A) $\sqrt[3]{9}, \sqrt[9]{5}, \sqrt[3]{7}$

(B) $\sqrt[9]{5}, \sqrt[3]{9}, \sqrt[3]{7}$

(C) $\sqrt[9]{5}, \sqrt[3]{7}, \sqrt[3]{9}$

(D) $\sqrt[3]{7}, \sqrt[9]{5}, \sqrt[3]{9}$

20. If $3-2\sqrt{2} = a+b\sqrt{2}$, then b = ____.

(A) $\sqrt{2}$

(B) 3

(C) -2

(D) 2

21. The descending order of $\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$ is:

(A) $\sqrt{5}, \sqrt[3]{7}, \sqrt[4]{6}$

(B) $\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

(C) $\sqrt[3]{7}, \sqrt{5}, \sqrt[4]{6}$

(D) $\sqrt[3]{7}, \sqrt[4]{6}, \sqrt{5}$

22. Express the surd $\frac{\sqrt[3]{5}}{\sqrt[3]{6}}$ with rational denominator is:

(A) $\frac{\sqrt[3]{180}}{6}$ (B) $\frac{\sqrt[3]{180}}{4}$

(C) $\frac{\sqrt[6]{180}}{3}$ (D) $\frac{\sqrt[3]{108}}{6}$

23. Conjugate of the $\sqrt{3}-11$ is:

(A) $\sqrt{3}+11$

(B) $3-\sqrt{11}$

(C) $11-\sqrt{3}$

(D) $11+\sqrt{3}$

24. The conjugate of $25-2\sqrt{19}$ is:

(A) $25+2\sqrt{19}$

(B) $-25+2\sqrt{19}$

(C) $-25-2\sqrt{19}$

(D) $\sqrt{25}-2\sqrt{19}$

25. Rationalizing the denominator of $\frac{3}{2\sqrt{2}}$ is:

(A) $\frac{2\sqrt{2}}{3}$

(B) $\frac{3\sqrt{2}}{4}$

(C) $\frac{9\sqrt{2}}{4}$

(D) $\frac{3\sqrt{2}}{4\sqrt{2}}$

26. Express $\frac{9}{\sqrt{3}}$ with rational denominator:

(A) $9\sqrt{3}$

(B) $3\sqrt{3}$

(C) 27

(D) $12\sqrt{3}$

27. $3\sqrt{6}+\sqrt{216}=$_______.

(A) $3\sqrt{6}$

(B) $5\sqrt{6}$

(C) $9\sqrt{6}$

(D) $7\sqrt{6}$

28. $\sqrt[3]{6}\times\sqrt[3]{6}\times\sqrt[3]{6}=$ ______.

(A) 18 (B) 6

(C) 2 (D) 216

29. Divide: $\sqrt[3]{144}$ by $\sqrt[6]{4}$.

(A) $6\sqrt[3]{81}$

(B) $\sqrt[3]{24}$

(C) $\sqrt[6]{24}$

(D) $2\sqrt[6]{81}$

30. If $x=\sqrt{6}+\sqrt{5}$, then $x^2+\frac{1}{x^2}-2=$_.

(A) $2\sqrt{6}$ (B) $2\sqrt{5}$

(C) 24 (D) 20

31. $\sqrt[4]{625}\div\sqrt[3]{15625}=$ ______.

(A) $\frac{1}{25}$ (B) $\frac{1}{15}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

32. The rationalising factor of $\frac{1}{5\sqrt{2}}$ is ______.

(A) 5 (B) $\sqrt{2}$

(C) $\frac{1}{5\sqrt{2}}$ (D) $5\sqrt{2}$

33. Conjugate of $4\sqrt{11}-3\sqrt{7}$ is ____.

(A) $3\sqrt{7}-4\sqrt{11}$

(B) $4\sqrt{11}-3\sqrt{7}$

(C) $4\sqrt{11}+3\sqrt{7}$

(D) Doesn't exists

34. If the conjugate surd of $\sqrt{5}+9$ is $-\sqrt{x}+y$ then $(y-x)^3$ = ______.

(A) 64

(B) 62

(C) 16

(D) 81

Multi Correct Questions (M.C.Q)

35. Which of the following is / are correct?

(A) The product of two rationals is a rational

(B) The product of two irrationals must be an irrational

(C) The sum of a rational and an irrational is an irrational

(D) The product of a rational and an irrational is a rational.

36. Which of the following is a correct statement?

(A) The irrational number between a and b is $\frac{a+b}{2}$

(B) π is an irrational number

(C) The value of $\sqrt{2}$ up to five decimal places is 1.41421.

(D) Irrationals are neither terminating nor recurring decimals.

37. Which of the following is/are correct?

(A) $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number

(B) $\sqrt[3]{5}$ is a surd and $\sqrt[3]{5}$ is an irrational number

(C) π is an irrational number, but it is not a surd

(D) $3+\sqrt{2}$ is a rational number

38. Which of the following is/are correct?

(A) $\sqrt[4]{2} < \sqrt[4]{7}$

(B) $\sqrt[4]{7} < \sqrt[4]{5}$

(C) $\sqrt[5]{10} < \sqrt[5]{13}$

(D) $\sqrt[3]{3} > \sqrt[3]{5}$

39. $\left(\sqrt[6]{5}\right)\left(\sqrt[3]{2}\right)\left(\sqrt{3}\right)\left(\sqrt[12]{6}\right)$ = ______.

(A) $\sqrt[12]{1749600}$

(B) $\sqrt[3]{2} \times \sqrt[12]{109350}$

(C) $\sqrt[12]{177960}$

(D) $\sqrt[12]{2} \times \sqrt[12]{109350}$

40. If $\sqrt[3]{2}+\sqrt[3]{16}-\sqrt[3]{54}=k$ then ______.

(A) k is least whole number

(B) k is an integer

(C) k is least natural number

(D) k is rational number

Comprehension Passage (C.P.T)

Passage - i

Irrational numbers satisfy the commutative, associative and distributive but not closure with respect to addition and multiplication.

41. **If $a = 8+\sqrt{5}, b = 1-\sqrt{5}$, then a + b is ______.**

(A) Rational

(B) Irrational

(C) Surd

(D) Radical

42. **If $a = \sqrt{3}, b = 4+\sqrt{3}$, then a – b is ________ number.**

(A) Surd

(B) Irrational

(C) Rational

(D) Radical

43. **If $a = 3+\sqrt{7}, b = 3-\sqrt{7}$, then a × b is ______.**

(A) Surd

(B) Irrational

(C) Radical

(D) Rational

Passage - ii

A surd, expressed in the form $a\sqrt[n]{b}$, where 'b' is the least positive rational number, is called the simplest form of the given surd.

44. **The simplest form of $\sqrt[4]{112}$ is:**

(A) $2\sqrt[3]{7}$

(B) $3\sqrt[3]{7}$

(C) $2\sqrt[4]{7}$

(D) $3\sqrt[4]{7}$

45. **Which of the following surds is not in simplest form?**

(A) $\sqrt{102}$

(B) $\sqrt{116}$

(C) $\sqrt{110}$

(D) $\sqrt{118}$

46. **$\sqrt{80}$ in simplest form is equal to:**

(A) $4\sqrt{5}$ (B) $2\sqrt{20}$

(C) $8\sqrt{10}$ (D) $5\sqrt{16}$

Matrix Matching Type (M.M.T.)

I. Column- I

47. The number $\frac{3}{4}$ is a

48. $\sqrt{2}$ is a

49. The number $\frac{-12}{4}$ is a

50. The value of π is

Column -II

(A) Whole number

(B) Irrational number

(C) Positive Rational number

(D) Does not exist in Q

(E) Negative Rational number

II. Column-I | **Column-II**

Column-I	Column-II
51. $\sqrt{32}+\sqrt{50}+\sqrt{128}=$	(A) $3\sqrt[3]{5}$
52. $3\sqrt{50}-4\sqrt{8}+7\sqrt{18}=$	(B) 0
53. $2\sqrt[3]{40}+3\sqrt[3]{625}-4\sqrt[3]{320}=$	(C) $17\sqrt{2}$
54. $8\sqrt{45}-8\sqrt{20}+\sqrt{245}-3\sqrt{125}=$	(D) $28\sqrt{2}$
	(E) Integer

Assertion Reason Type (A.R.T.)

(A) Both Assertion(A) and Reason(R) are correct and reason(R) is the correct explanation of assertion(A).

(B) Both Assertion(A) and Reason(R) are correct but reason(R) is not the correct explanation of assertion(A).

(C) Assertion(A) is correct but Reason(R) is incorrect.

(D) Assertion(A) is incorrect but Reason(R) is correct.

55. Assertion: $3\sqrt{3}+5\sqrt{3}-\sqrt{27}=5\sqrt{3}$

Reason: $a\sqrt{d}+b\sqrt{d}-c\sqrt{d}=(a+b-c)\sqrt{d}$

56. Assertion: $2+\sqrt{6}$ is an irrational number.

Reason: Sum of a rational number and an irrational number is always an irrational number.

57. Assertion: $\sqrt[3]{2010}$ is a cubic surd.

Reason: A surd of order 3 is called quadratic surd.

Integer Type Question (I.T.Q.)

58. Cubic surd is a surd of order ________.

59. If $\frac{5\sqrt{5}}{3}=\sqrt{\frac{125}{k}}$ then k = ________.

60. If $5+\sqrt{7}$ and $5-\sqrt{m}$ are conjugate surds to each other then least possible value for 13 - m = ______.

61. If $\sqrt[3]{2}=\sqrt[n]{64}$, $\sqrt[9]{4}=\sqrt[n]{16}$, $\sqrt[6]{3}=\sqrt[n]{27}$

then $\frac{n}{9}=$ ______.

Previous Question (P.Q.)

62. A rational number can be expressed as a terminating decimal if the denominator has factors: [NSTSE]

(A) 2 or 5

(B) 2, 3 or 5

(C) 3 or 5

(D) None of these

63. $p = 5+2\sqrt{6}$ and $q = \frac{1}{p}$ then, p^2+q^2 is: [ntse]

(A) 49

(B) 98

(C) 100

(D) None of these

64. Expressing $0.\overline{34} + 0.\overline{34}$ as a single decimal, we get: [ntse]

(A) $0.67\overline{88}$

(B) $0.6\overline{89}$

(C) $0.6\overline{878}$

(D) $0.6\overline{87}$

65. If $x = \frac{1}{1+\sqrt{2}}$, then the value of $x^2+2x^{+}3$ is: [ntse]

(A) 3

(B) 0

(C) 4

(D) 1

66. If $\sqrt{6}$ = 2.449, then the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$ is close: [ntse]

(A) 1.225

(B) 0.816

(C) 0.613

(D) 2449

67. The decimal expansion of π is: [ntse]

(A) a whole number

(B) terminating

(C) non-terminating but repeating

(D) non-terminating non-repeating

68. The sum of rational and irrational number is: [ntse]

(A) Rational

(B) Irrational

(C) Zero

(D) Integers

69. Which of the following can be expressed as the sum of square of two positive integers. as well as three positive integers? [ntse]

(A) 75

(B) 192

(C) 250

(D) 100