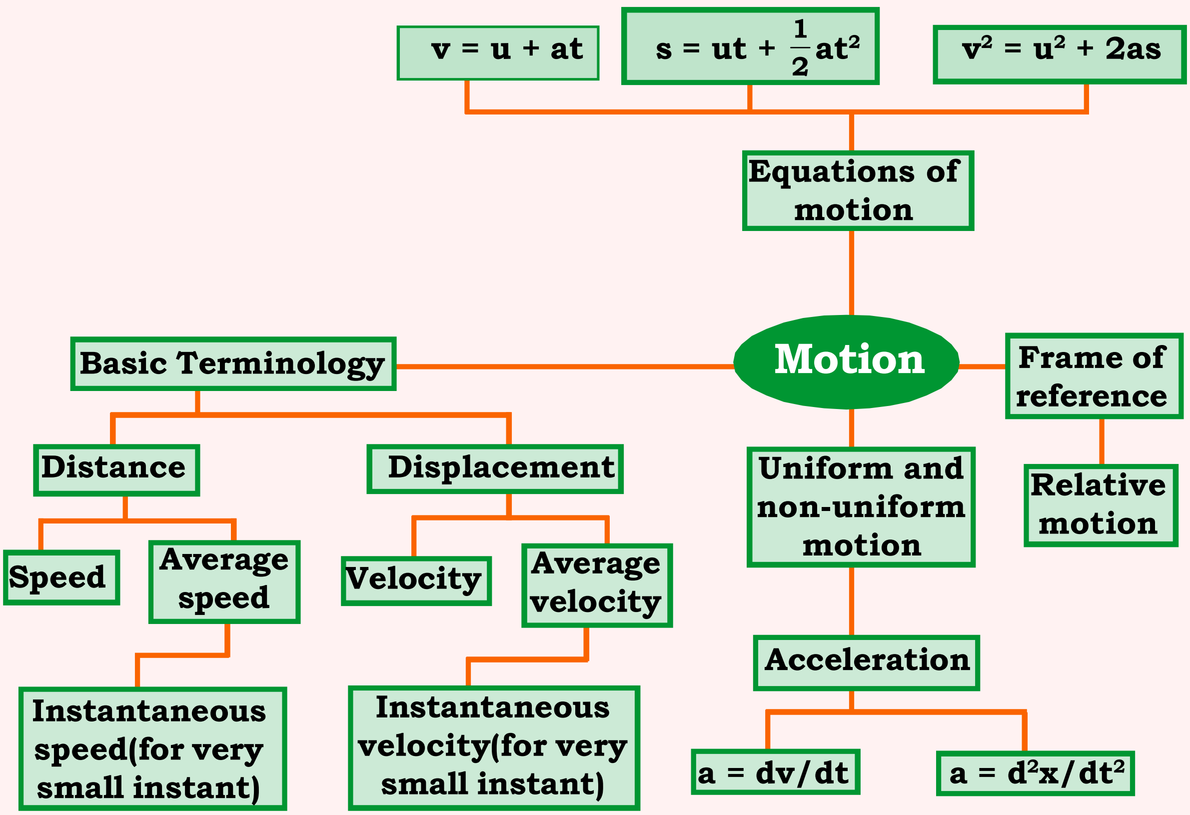
Chapter 1

MOTION

Albert Abraham: German-American Physicist, inventor and experimenter devised Michelson's interferometer with the help of which, in association with Morley, he tried to detect the motion of earth with respect to ether but failed.

However, the failed experiment stirred the scientific world to reconsider all old theories and led to a new world of physics. He developed a technique for increasing the resolving power of telescopes by adding external mirrors.

CONCEPT MAP



CONCEPT 1.1

Mechanics:

Motion is caused by force. The branch of physics which deals with the effect of forces on objects is called mechanics.

Mechanics can be classified into two categories -

(i) Statics and (ii) Dynamics

Statics:

It is the branch of mechanics which deals with objects at rest under the action of forces.

Dynamics:

It is the branch of mechanics which deals with objects under motion.

Dynamics can again be classified into two categories -

(i) Kinematics and (ii) Kinetics.

Kinematics:

It deals with the motion of objects without considering about the cause of motion.

Kinetics:

It deals with the motion of objects considering the cause of their motion.

A point object:

An object is said to be a point object if its dimensions (i.e. length, breadth and thickness etc.) are negligible as compared to the distance travelled by it.

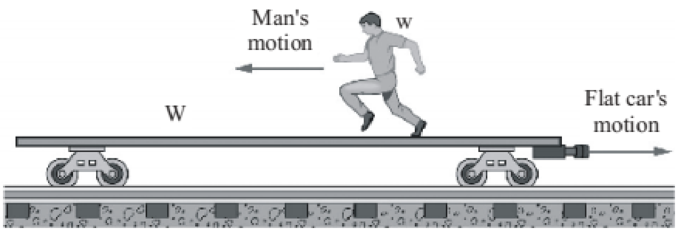
Rest and Motion:

Rest: An object is said to be at rest if it does not change its position with respect to its surroundings with the passage of time.

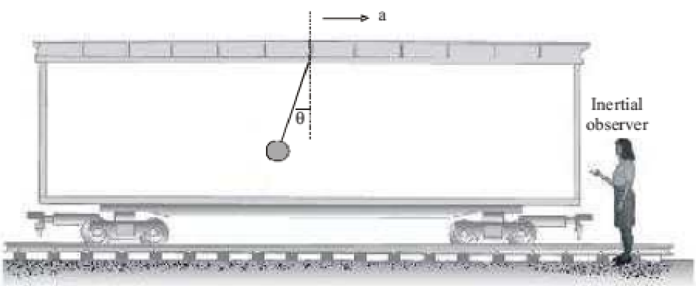
Motion: A body is said to be in motion if its position changes continuously with respect to the surroundings (or with respect to an observer) with the passage of time. We know that earth is rotating about its axis and revolving around the sun. The stationary objects like your classroom, a tree and the lamp posts etc. do not change their position with respect to each other i.e., they are at rest. Although earth is in motion, to an observer situated outside the earth say in a spaceship, your classroom, trees etc. would appear to be in motion. Therefore, all motions are relative. There is nothing like absolute motion. If you move with book in your hand, book is not moving with respect to you.

Observe like a science student:

To the passengers in a moving bus or train, trees, buildings and people on the roadsides observe that the bus or the train and its passengers are moving in the forward direction. At the same time, each passenger in a moving bus or train finds that fellow passengers are not moving, as the distance between them is not changing. These observations tell us that the motion is relative. If you will observe the man moving on moving flat car from ground your observation will be different from what a man himself will observe. Similarly, if you will observe pendulum in moving car from ground your observation will be different from what person inside car will observe.



Man and flat car in relative motion with respect to an observer on the road.



(a) An observer outside the car observing the motion of a pendulum.

Scalars & Vectors:

Physical quantities are classified into scalars and vectors.

Scalar quantities:

1. Physical quantities having only magnitude are called scalars.
2. **Examples:** Distance, speed, mass, time, temperature, density, work, energy, power etc.

Vector Quantities:

1. Physical quantities having both magnitude and direction and which obey the laws of vector addition are called vectors.
2. **Examples:** Displacement, velocity, acceleration, linear momentum, force etc.

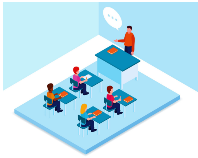
3. Any directed line segment is a vector which has three characteristics viz; (namely) 1) support (base) 2) length (magnitude) and 3) sense (direction). Moreover a vector should follow certain geometric laws.

4. The direction from A to B is denoted by $\overrightarrow{AB}$. The direction from B to A is denoted by $\overrightarrow{BA}$.

5. $\left|\overrightarrow{AB}\right|$ implies modulus of the vector or length of the vector or magnitude of vector.

6. If a physical quantity has both magnitude and direction then it does not always imply that it is a vector.

7. For it to be a vector i) It should be resolved into mutually perpendicular directions and ii) It should obey the certain geometric laws of vector addition.



CLASSROOM DISCUSSION QUESTIONS CDQ 1.1

1. Which branch of mechanics deals with objects at rest under the action of forces?

(A) Dynamics (B) Kinematics

(C) Mechanics (D) Statics

2. What category of mechanics deals with the motion of objects without considering the cause of motion?

(A) Kinetics (B) Statics

(C) Kinematics (D) Dynamics

3. What is the characteristic of a scalar quantity?

(A) It has both magnitude and direction

(B) It obeys the laws of vector addition

(C) It has only magnitude

(D) It has neither magnitude nor direction

4. Which of the following is a vector quantity?

(A) Distance

(B) Speed

(C) Displacement

(D) Time

5. How is the direction from point A to point B denoted in vector notation?

(A) BA (B) AB

(C) AB (D) AB

6. What characteristic must a physical quantity have to be considered a vector?

(A) It must be resolved into mutually perpendicular directions

(B) It must have magnitude only

(C) It must have no direction

(D) It must follow certain geometric laws

7. What is the branch of mechanics that deals with objects under motion?

(A) Statics (B) Dynamics

(C) Kinetics (D) Kinematics

8. Which of the following is an example of a scalar quantity?

(A) Displacement

(B) Velocity

(C) Mass

(D) Force

9. What term is used to describe an object if its dimensions are negligible compared to the distance traveled by it?

(A) Point object

(B) Scalar object

(C) Vector object

(D) Magnitude object

10. What observation about motion can be derived from the fact that all motions are relative?

(A) Motion is absolute

(B) Motion is only observed by stationary objects

(C) Motion depends on the observer's frame of reference

(D) Motion is constant regardless of perspective

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CONCEPT 1.2

Distance:

The actual length of the path travelled by the particle in a given interval of time is called distance.

Its SI unit is meter (m) and its C.G.S. unit is centimetre (cm).

It is a scalar quantity.

The dimensional formula of distance is $[M^0L^1T^0]$.

Displacement:

The shortest path covered by an object from initial point to the final point is called displacement.

Its SI unit is metre (m) and its C.G.S. unit is centimetre (cm).

It is a vector quantity.

Its magnitude is equal to the length between the initial position and final position Its direction is from the initial position to final position.

Speed:

The distance travelled by a body per unit time is called speed.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

It's basic unit (or) SI unit is m/s and the CGS unit is cm/s.

Speed is a scalar quantity.

The dimensional formula of speed is $[M^0L^1T^{-1}]$.

The other units of speed are kmph (km/h).

Velocity:

The rate of change of displacement is called "Velocity".

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

Its basic unit (or) SI unit is m/s and the CGS unit is cm/s.

Velocity is a vector quantity.

The dimensional formula of velocity is $[M^0L^1T^{-1}]$.

Change of velocity:

1. Velocity of a particle changes when
   i) Its magnitude is changed (or)
   ii) Its direction is changed (or)
   iii) Both magnitude and direction are changed.

2. Change in velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ where $\vec{v}_f$ is final velocity $\vec{v}_i$ is initial velocity. Change in velocity is absolute in an inertial frame. So it does not depends on the observer.

Acceleration:

1. The change in velocity in unit time is called acceleration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}} = \boxed{\vec{a} = \frac{\overrightarrow{\Delta v}}{t}} = \frac{(\vec{v} - \vec{u})}{t}$$

   Here, $\vec{u}$ is the initial velocity and $\vec{v}$ is the final velocity.

2. Acceleration is a vector.
3. The direction of acceleration will be in the direction of change in velocity.
4. The dimensional formula of acceleration is $[M^0L^1T^{-2}]$.
5. Its SI unit is metre/second^2 abbreviated as m/s^2.

Average Acceleration:

If the velocity of a particle remains constants as time passes, we say that it is moving with

uniform velocity. If the velocity changes with time, it is said to be accelerated. The acceleration is

the rate of change of velocity.

Thus, $\vec{a}_{avg} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

Instantaneous acceleration:

The acceleration of the body at a particular instant is called its "Instantaneous acceleration".

It is given by $a = \dfrac{dv}{dt}$

Uniform Acceleration or constant acceleration:

If the velocity of the body change by equal amounts in equal intervals of time, then it is said

to be in uniform acceleration or constant acceleration. If the velocity of a uniformly accelerating

body changes from u to v, then its acceleration is given by

$$a = \frac{v - u}{t} \text{ or } v = u + at$$

Non-uniform acceleration:

If the velocity of the body changes by unequal amounts in equal intervals of time, then it is

said to be in non-uniform acceleration.

Deceleration:

If the speed of a particle decreases with time, we say that it is decelerating or retardation.

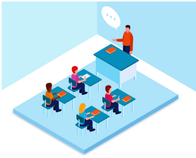
The acceleration of the body is opposite to that of velocity, and then the body decelerates. It is

denoted by the symbol '-a'.

Equations of Uniformly Accelerated Motion:

There are three equations which establish the relationship of uniform acceleration 'a', with initial velocity 'u', the final velocity 'v', the time in seconds 't' and the distance covered 'S'.

1. $v = u + at$ **2.** $S = ut + \frac{1}{2}at^2$ **3.** $v^2 - u^2 = 2aS$



CLASSROOM DISCUSSION QUESTIONS CDQ 1.2

1. Which physical quantity is defined as the length of the path traveled by an object?

(A) Velocity

(B) Distance

(C) Displacement

(D) Acceleration

2. What is the SI unit of distance?

(A) Centimeter (cm)

(B) Meter per second (m/s)

(C) Meter (m)

(D) Kilometer per hour (km/h)

3. What is the dimensional formula of distance?

(A) $[M^0L^1T^{-1}]$ (B) $[M^0L^0T^1]$

(C) $[M^0L^1T^0]$ (D) $[M^1L^0T^{-1}]$

4. What physical quantity is measured in meter per second (m/s)?

(A) Speed

(B) Acceleration

(C) Displacement

(D) Distance

5. Which of the following quantities is a vector?

(A) Distance

(B) Speed

(C) Acceleration

(D) Time

6. What is the dimensional formula of velocity?

(A) $[M^0L^0T^1]$ (B) $[M^0L^1T^{-1}]$

(C) $[M^0L^1T^0]$ (D) $[M^1L^0T^{-1}]$

7. What does acceleration represent?

(A) Change in position with time

(B) Rate of change of velocity with time

(C) Change in speed with time

(D) Rate of change of displacement with time

8. Which equation establishes the relationship between velocity, acceleration, and time for uniformly accelerated motion?

(A) $v = u + at$

(B) $v^2 - u^2 = 2aS$

(C) $S = ut + 1/2\ at^2$

(D) $a = (v - u)/t$

9. What is the instantaneous acceleration of an object?

(A) The acceleration measured over a long period of time

(B) The acceleration when the velocity is constant

(C) The acceleration at a specific moment in time

(D) The average acceleration over a period of time

10. When does a particle experience deceleration?

(A) When its velocity is constant

(B) When its speed increases with time

(C) When its speed decreases with time

(D) When its velocity is zero

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.3

Acceleration Due to Gravity:

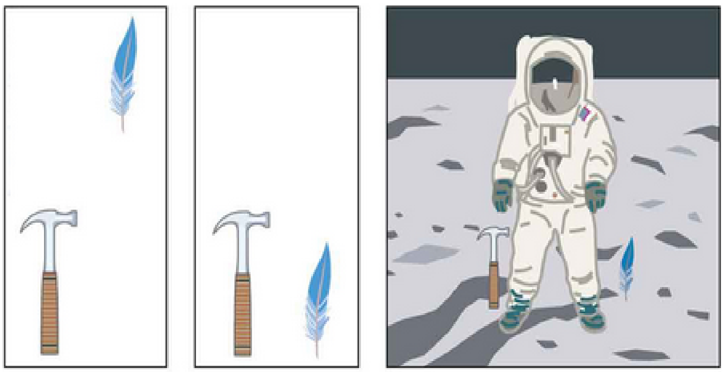
The uniform acceleration of a freely falling body towards the centre of earth due to earth's gravitational force is called "Acceleration due to gravity".

i) It is denoted by 'g'.

ii) Its value is constant for all bodies at a given place.

iii) It is independent of size, shape, material, constitution (hollow or solid), nature of the body.

iv) If air resistance is ignored, all the bodies as light as a feather to a heavy metal sphere, dropped simultaneously from the same height hit the floor at the same time because all the bodies have same acceleration due to gravity.



v) Its value changes from place to place on the surface of the earth.

vi) It has maximum value at the poles of the earth. The value is nearly 9.83 m/s^2.

vii) It has minimum value at the equator of the earth. The value is nearly 9.78 m/s^2.

viii) The average value of 'g' on the earth surface is 9.8 m/s^2.

ix) On the surface of moon, g = 1.67 m/s^2.
On the surface of sun, g = 27.4 m/s^2.

x) The acceleration due to gravity of a body is always directed downwards towards the centre of the earth, whether the body is projected upwards or downwards.

xi) When a body is falling towards the earth, its velocity increases and 'g' is positive.

xii) When a body is projected upwards, its velocity decreases and 'g' is negative.

xiii) The acceleration due to gravity at the centre of earth is zero.

Equations of motion for freely falling body:

Motion of all the dropped bodies falling towards the Earth when air resistance is ignored, is known as free fall.

For a freely falling body,

u = 0 m/s, a = + g

v = u + gt

Then the equations of motion become

1) $v = u + at \Rightarrow v = gt$

2) $s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}gt^2$

3) $v^2 - u^2 = 2as \Rightarrow v^2 = 2gs$

4) $s_n = u + a\left(n - \frac{1}{2}\right) \Rightarrow s_n = g\left(n - \frac{1}{2}\right)$

Note:

1) For a freely falling body, the ratio of distances travelled in 1 second, 2 seconds, 3 seconds, 4 seconds . . . . . . = 1: 4: 9: 16 . . . . so on.

2) For a freely falling body, the ratio of distances travelled in successive seconds = 1: 3: 5: 9 . . . . . . .so on.

3) In uniform accelerated motion, the distance travelled in every second increases by an amount equal to the magnitude of acceleration.

Equations of Motion for vertically projected upwards body:

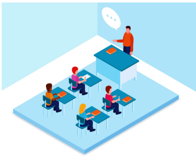
For a body projected vertically upwards, a = - g (since velocity, acceleration vectors are opposite).

1) $v = u + at \Rightarrow v = u - gt$

2) $s = ut + \frac{1}{2}at^2 \Rightarrow s = ut - \frac{1}{2}gt^2$

3) $v^2 - u^2 = 2as \Rightarrow v^2 - u^2 = -2gs$

4) $s_n = u + a\left(n - \frac{1}{2}\right) \Rightarrow s_n = u - g\left(n - \frac{1}{2}\right)$



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.3

1. What is the acceleration experienced by a freely falling body towards the center of the Earth called?

(A) Centripetal acceleration
(B) Gravitational acceleration
(C) Frictional acceleration
(D) Angular acceleration

2. Which of the following statements about acceleration due to gravity is correct?

(A) It depends on the size and shape of the object.
(B) Its value is highest at the equator of the Earth.
(C) It is independent of the material and constitution of the body.
(D) It has a constant value of 9.83 m/s^2 everywhere on Earth's surface.

3. At which location on Earth's surface does the acceleration due to gravity have its maximum value?

(A) At the equator
(B) At the poles
(C) At the Tropic of Cancer
(D) At the Tropic of Capricorn

4. What is the average value of acceleration due to gravity on the surface of the Earth?

(A) 9.67 m/s^2 (B) 9.78 m/s^2
(C) 9.8 m/s^2 (D) 10.2 m/s^2

5. What is the acceleration due to gravity at the center of the Earth?

(A) Zero (B) 9.8 m/s^2
(C) 9.83 m/s^2 (D) 1.67 m/s^2

6. In the equations of motion for a freely falling body, what does 'u' represent?

(A) Final velocity
(B) Acceleration
(C) Initial velocity
(D) Time

7. What is the value of acceleration for a freely falling body in the equations of motion?

(A) Zero
(B) Positive
(C) Negative
(D) Depends on the height of the fall

8. What is the value of acceleration for a vertically projected upwards body in the equations of motion?

(A) Zero
(B) Positive
(C) Negative
(D) Equal to the velocity

9. Which of the following ratios represents the distances traveled by a freely falling body in successive seconds?

(A) 1 : 2 : 3 : 4
(B) 1 : 3 : 5 : 7
(C) 1 : 4 : 9 : 16
(D) 1 : 5 : 9 : 13

10. What happens to the velocity of a body projected vertically upwards as it moves upwards?

(A) It increases
(B) It remains constant
(C) It decreases
(D) It fluctuates

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

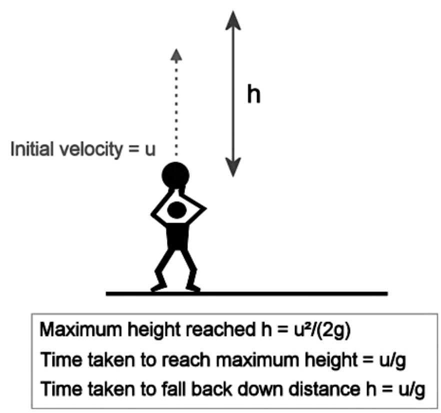
CONCEPT 1.4

Motion parameters of a body projected vertically upwards:

Maximum height:

For a body projected upwards, the maximum vertical displacement from ground about which its velocity is zero is called its maximum height.

Let a body be projected vertically upwards with an initial velocity 'u'.



Initial velocity, $u = u$ m/s

Final velocity, $v = 0$

Acceleration due to gravity = $-g$ m/s^2

Let the maximum height reached by the body be 'H_{max}'.

Then using the formula

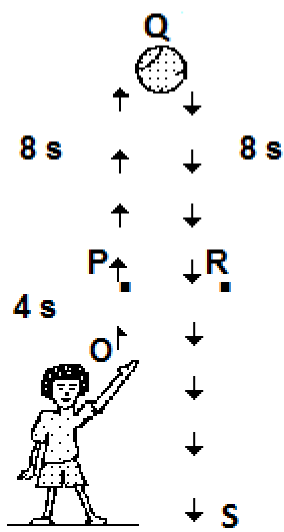
$v^2 - u^2 = 2gh.$

$\Rightarrow 0 - u^2 = -2gH_{max}$

$$\Rightarrow H_{max} = \frac{u^2}{2g}$$

Time of ascent:

For a body projected upwards the time to reach the maximum height is called time of ascent.



Let a body be projected vertically upwards with an initial velocity 'u'.

Initial velocity = u m/s

Acceleration of the body = $-g$ m/s^2

Final velocity of the body projected upwards at its maximum height = $v = 0$

Let the time of ascent be 't_a'

Now using formula

$v = u - gt$

$$\Rightarrow 0 = u - gt_a$$

$$\Rightarrow u = gt_a$$

$$\Rightarrow t_a = \frac{u}{g}$$

Time of descent:

For a body projected upwards, the time to travel from maximum height to the point of projection on ground is called time of descent.

Expression:

Let a body be projected vertically upwards with an initial velocity 'u'.

Step 1: For upward motion

We know that

$v^2 - u^2 = 2gh.$

$$\Rightarrow 0 - u^2 = -2gH_{max}$$

$$\Rightarrow H_{max} = \frac{u^2}{2g}$$

Step 2: For downward motion

We know that

$$s = ut + \frac{1}{2}at^2$$

For a body projected upwards at maximum height the velocity becomes zero. This will be considered as initial velocity during downward motion.

Here initial velocity = 0, a = + g, t = t_d, s = H_{max}.

$$\Rightarrow H_{max} = 0 + \frac{1}{2}gt_d^{\,2}$$

$$\Rightarrow \frac{u^2}{2g} = \frac{1}{2}gt_d^{\,2} \qquad \Rightarrow t_d^2 = \frac{u^2}{g^2}$$

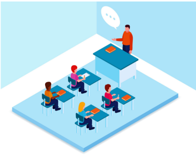
$$\Rightarrow t_d = \frac{u}{g}$$

Time of flight:

For a body projected vertically upwards the sum of time of ascent and time of descent is called time of flight (T). It is the total time for which the body remains in air.

Time of flight = Time of ascent + Time of descent

It is given by T = $t_a + t_d$ $= \frac{u}{g} + \frac{u}{g} = \frac{2u}{g}$



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.4

1. What is the maximum height reached by a body projected vertically upwards with an initial velocity 'u'?

(A) $\frac{u^2}{2g}$ (B) $\frac{u}{2g}$ (C) $\frac{2u}{g}$ (D) $\frac{u^2}{g}$

2. Which of the following parameters is used to calculate the time taken by a body to reach the maximum height during its upward motion?

(A) Final velocity
(B) Initial velocity
(C) Acceleration due to gravity
(D) Time of ascent

3. What is the expression for the time taken by a body to descend from its maximum height to the ground after being projected vertically upwards?

(A) $\frac{2H_{max}}{g}$ (B) $\frac{H_{max}}{g}$ (C) $\frac{H_{max}}{2g}$ (D) $\frac{H_{max}}{2}$

4. What is the total time for which a body remains in the air after being projected vertically upwards, including both the ascent and descent times?

(A) Time of flight
(B) Time of ascent
(C) Time of descent
(D) Time of projection

5. How is the maximum height of a body related to its initial velocity and acceleration due to gravity?

(A) $H_{max} = \frac{u}{2g}$ (B) $H_{max} = \frac{2u}{g}$

(A) $H_{max} = \frac{u^2}{2g}$ (A) $H_{max} = \frac{u^2}{g}$

6. Which equation is used to calculate the time of ascent for a body projected vertically upwards?

(A) $v = u - gt$ (B) $v^2 - u^2 = 2gh$

(C) $s = ut + \frac{1}{2}gt^2$ (D) $v = u + gt$

7. What is the velocity of a body at the maximum height during its upward motion?

(A) Zero
(B) Equal to the initial velocity
(C) Equal to the acceleration due to gravity
(D) Twice the initial velocity

8. What is the expression for the time of descent for a body projected vertically upwards?

(A) $\frac{H_{max}}{g}$ (B) $\frac{H_{max}}{2g}$ (C) $\frac{2H_{max}}{g}$ (D) $\frac{H_{max}}{2}$

9. What is the total displacement covered by a body during its upward and downward motion if projected vertically upwards?

(A) $\frac{2H_{max}}{g}$ (B) $\frac{H_{max}}{g}$

(C) $\frac{H_{max}}{2g}$ (D) $\frac{H_{max}}{2}$

10. What is the sum of the time of ascent and time of descent for a body projected vertically upwards?

(A) Time of flight
(B) Time of ascent
(C) Time of descent
(D) Time of projection

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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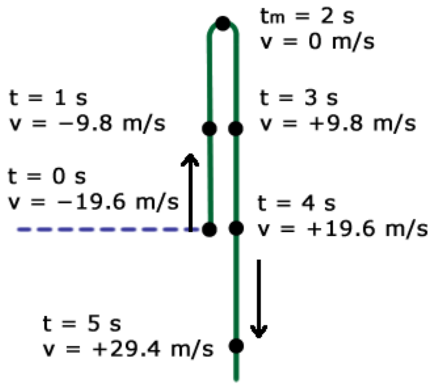
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.5

Velocity of the body on reaching the point of projection:

Let a body be projected vertically upwards with an initial velocity 'u'.

The body reaches the point of projection once again after the time of flight (T).



We know that

$v = u + at$

Here $a = -g$; $t = T = 2u / g$; $v = V_{striking}$

$\Rightarrow \quad V_{striking} = u - gT$

$\Rightarrow \quad V_{striking} = u - g(2u / g)$

$\Rightarrow \quad V_{striking} = u - 2u$

$\Rightarrow \quad V_{striking} = -u$

So, the body reaches the point of projection with the same speed of projection but in opposite direction.

Velocity at a given height:

Let a body be projected vertically upwards with an initial velocity u.

The body reaches a height 'h' after certain time 't'.

We know that

$v^2 = u^2 + 2as$

Here $v = V$; $u = u$; $a = -g$; $s = h$

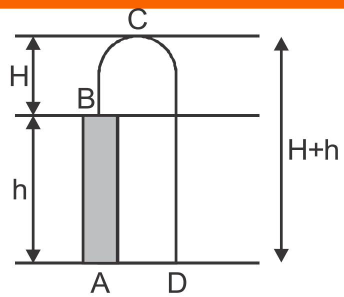
$\Rightarrow \quad v^2 = u^2 - 2gh$

$\Rightarrow \quad v = \sqrt{u^2 - 2gh}$

NOTE:

1. A body can have zero velocity and non-zero acceleration at an instant. For example, at highest point of vertically projected body the velocity vector is null vector, but acceleration vector is vertically downwards.
2. Distance covered by the body projected vertically up in the 1st second of its upward journey is equal to the distance covered by it in the last second of its downward journey.
   $s = u - g/2$
3. Distance covered by the body projected vertically up in the last second of its upward journey is equal to the distance covered by it in the 1st second of its downward journey.
   $s = g/2$

Motion of a body thrown vertically up from top of a tower:



AB is a tower of height h. A body is projected vertically upwards with an initial velocity 'u' from the top 'B' of the tower. The body travels upwards, reaches the highest point C and starts falling downwards thereafter. Finally, the body reaches the point D lying in the horizontal plane passing through foot of the tower. Here the direction of projection (upwards) is considered as positive and the opposite direction (i.e., downwards) is taken negative.

The upward displacement of the body;

The downward displacement of the body;

CD = – (h + H)

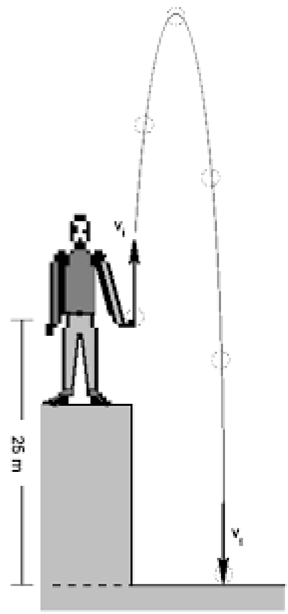
The net displacement of the body;

BD = +H – (h + H) = – h

The total time of travel from B to D through C = t (say)

$\therefore$ For the motion of the body

Displacement (s)	=	– h
Acceleration (a)	=	– g (Starting upwards)
Time of travel (t)	=	t
Initial velocity (u)	=	u



From the equation of motion $s = ut + \frac{1}{2}at^2$

$\Rightarrow -h = ut - \frac{1}{2}gt^2 \qquad \Rightarrow h = -ut + \frac{1}{2}gt^2$

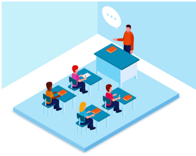
$\therefore$ The height of tower is given by $h = \frac{1}{2}gt^2 - ut$.

Note:

This is a quadratic equation in time $\left(gt^2 - 2ut - 2h = 0\right)$.

Comparing with the standard quadratic equation $ax^2 + bx + c = 0$

We get $t = \frac{u \pm \sqrt{u^2 + 2gh}}{g}$



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.5

1. What is the velocity of a body when it reaches the point of projection after completing its upward journey?

(A) Zero

(B) Equal to its initial velocity

(C) Equal to its acceleration due to gravity

(D) Equal to negative of its initial velocity

2. How can the velocity of a body at a given height be calculated when it's projected vertically upwards?

(A) $v = u^2+2gh$ (B) $v = u^2-2gh$

(C) $v = u+2gh$ (D) $v = u-2gh$

3. At what point during its motion does a vertically projected body have a null velocity but non-zero acceleration?

(A) Start of motion

(B) Highest point

(C) Midway

(D) End of motion

4. What is the distance covered by a body projected vertically upwards in the last second of its upward journey?

(A) $u-g$ (B) $u+g$

(C) g (D) u

5. What is the upward displace-ment of a body thrown vertically up from the top of a tower?

(A) h (B) H

(C) h + H (D) –h

6. What is the downward displacement of a body thrown vertically up from the top of a tower?

(A) h (B) H

(C) h + H (D) –(h+H)

7. What is the net displacement of a body thrown vertically up from the top of a tower?

(A) h (B) H

(C) h + H (D) –h

8. What is the total time taken by a body thrown vertically up from the top of a tower to reach the horizontal plane passing through the foot of the tower?

(A) t (B) 2t

(C) $\frac{2u}{g}$ (D) $\frac{u^2}{g}$

9. How is the height of the tower related to the time taken by a body thrown vertically up from its top to reach the horizontal plane?

(A) $h=ut+\frac{1}{2}gt^2$ (B) $h=ut-\frac{1}{2}gt^2$

(C) $h=u+\frac{1}{2}gt^2$ (D) $h=u^2+2gt$

10. Which equation represents the height of the tower from which a body is thrown vertically upwards?

(A) $u^2-2gh = 0$ (B) $u^2-2gh = h$

(C) $u^2-2gh = -h$ (D) $u^2+2gh = -h$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 1.6

Solved problems:

Problem 1: A helicopter is ascending vertically with a speed of 8.0 ms^{-1}. At a height of 120 m above the earth, a package is dropped from a window. How much time does it take for the package to reach the ground?

Solution: u = 8 m/s; height h = 120m $\Rightarrow$ s = – 120 m

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad -120 = 8t - \frac{1}{2}\times 9.8\times t^2$$

$4.9t^2 - 8t - 120 = 0$ (This is in the form of quadratic equation $ax^2 + bx + c = 0$)

$$t = \frac{8 \pm \sqrt{64 + 4\times 4.9\times 120}}{2\times 4.9} \qquad \left(\because t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{8 \pm \sqrt{64 + 2352}}{9.8} = \frac{8 \pm \sqrt{2416}}{9.8} = \frac{8 \pm 49.15}{9.8}$$

$$t = \frac{8 \pm 49.15}{9.8} = \frac{57.15}{9.8}$$ (only positive value) = 5.83 sec.

Problem 2: Two balls are dropped to the ground from different heights. One ball is dropped 2.0 s after the other, but both strike the ground at the same time 5.0 s after the 1st is dropped. (a) What is the difference in the heights from which they were dropped? (b) From what height was the first ball dropped?

Solution: (a) For the first ball s = h_1; v = 0; t = 5 sec

$$s = ut + \frac{1}{2}at^2$$

$$h_1 = 0\times 5 + \frac{1}{2}9.8\times 5^2 = 122.5 \text{ m}$$

For the second ball s = h_2; u = 0; t = 3 sec

$$\therefore h_2 = \frac{1}{2}gt^2 = \frac{1}{2}\times 9.8\times 9 = 4.9\times 9 = 44.1 \text{ m}$$

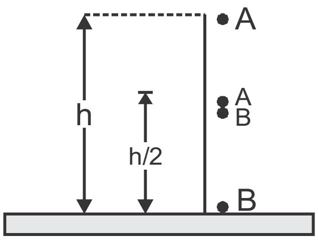
Difference in heights h = $h_2 - h_1$ = 122.5 – 44.1 = 78.4 m

(b) The first ball was dropped from a height of h_1 = 122.5 m

Problem 3: A stone A is dropped from rest from a height h above the ground. A second stone B is simultaneously thrown vertically up from a point on the ground with velocity u. The line of motion of both the stones is same. Find if the value of v which would enable the stone B to meet the stone A midway between their initial positions.

Solution: Time of travel of each stone = Distance travelled by each stone = $\frac{h}{2}$

For stone A, $\frac{h}{2}=\frac{1}{2}gt^2$ i.e., $t=\sqrt{\frac{h}{g}}$



For stone B, $\frac{h}{2}=ut-\frac{1}{2}gt^2=u\sqrt{\frac{h}{g}}-\frac{1}{2}g\left(\frac{h}{g}\right)$

$\Rightarrow \frac{h}{2}=u\sqrt{\frac{h}{g}}-\frac{h}{2}$ or $u\sqrt{\frac{h}{g}}=h$

$\therefore u=h/\sqrt{\frac{h}{g}}=\sqrt{h}\times\sqrt{h}/\sqrt{\frac{h}{g}}=\sqrt{gh}$

Problem 4: A particle is thrown vertically upwards. If its velocity at half of the maximum height is 10 m/s then what is the maximum height attained by it? (Take g = 10 m/s²)

Solution: Let particle thrown with velocity u and its maximum height is H

then $H=\frac{u^2}{2g}$

When particle is at a height $\frac{H}{2}$, then its speed is 10 m/s

From equation $v^2-u^2=2gh$

$10^2=u^2-2g\left(\frac{H}{2}\right)$; $10^2=u^2-2g\left(\frac{u^2}{4g}\right)$

$10^2=u^2-\frac{u^2}{2}$

$u^2=200\,m/s$

Maximum height $H=\frac{u^2}{2g}=\frac{200}{2\times 10}=10m$

C.D.F. CONCEPTS, DEFINITIONS AND FORMULAE

1. **Vector:** Quantities which require both magnitude and direction for their complete specification are called vectors.
2. **Representation of a vector:** A vector can be conveniently represented by a straight line with an arrowhead. The length of the vector represents its magnitude, and the arrowhead indicates its direction.
3. Quantities which require only magnitude for their complete specifications and having no direction associated with them are called scalar quantities.
4. We know that all objects, when dropped, fall towards the Earth with constant acceleration due to gravitational force of the earth. This is called acceleration due to gravity (g).
5. Objects in motion under the influence of gravity are said to be in free fall.
6. **Equations of motion for freely falling body:**

$$v = u + gt \; ; \qquad h = ut + \frac{1}{2}gt^2 \; ; \qquad v^2 - u^2 = 2gh$$

7. **Equations of Motion of an object vertically Projected Upwards from the Ground:**

$$v = u - gt \; ; \qquad h = ut - \frac{1}{2}gt^2 \; ; \qquad v^2 - u^2 = -2gh$$

8. When a body is projected vertically upwards with an initial velocity 'u'; the maximum height reached by the body is given by $h = \frac{u^2}{2g}$
9. Time of ascent of a vertically projected body is given by $t_a = \frac{u}{g}$
10. Time of descent of a vertically projected body is given by $t_d = \sqrt{\frac{2h}{g}} = \frac{u}{g}$
11. Time of flight of a vertically projected body is given by $t_a = \frac{2u}{g}$
12. Velocity at a given height for a vertically projected body is given by $v = \pm\sqrt{u^2 - 2gh}$
13. For a freely falling body ratio of the distances travelled after 1^{st} t seconds, next t seconds, next 't' seconds, ... is 1: 3: 5: ....

ADVANCED WORKSHEET

Single Correct Answer Type (S.C.A.T)

1. All the bodies near the earth surface have same acceleration due to gravity irrespective of their:

(A) Shape (B) Size

(C) Mass (D) All of these

2. The objects that move under the influence of gravity without any other external force acting on them are called:

(A) Freely falling bodies

(B) Vertically projected upwards bodies

(C) Both (A) & (B)

(D) None of these

3. The maximum height reached by a body projected upwards with an initial velocity 'u' is given by:

(A) $h=\frac{u}{2g}$ (B) $h=\frac{u}{2g^2}$

(C) $h=\frac{u^2}{2g^2}$ (D) $h=\frac{u^2}{2g}$

4. At any height a particle can have:

(A) Two velocities of same magnitude but opposite direction

(B) Two velocities of different magnitude same direction

(C) Two velocities of same magnitude and same direction

(D) two velocities of different magnitudes and different directions

5. Which of the following options is/are correct?

(A) If the upward direction is taken as positive, then g will be negative

(B) If the upward direction is taken as negative, then g will be positive

(C) Both (A) & (B)

(D) None of the above

6. For a physical quantity to be a vector, it should be resolved into mutually _____ directions.

(A) Parallel

(B) Perpendicular

(C) Both (A) & (B)

(D) None of these

7. The numerical ratio of displacement to distance for a moving object is:

(A) Always less than 1

(B) Always equal to 1

(C) Always more than 1

(D) Equal to less than 1

8. The numerical ratio of average velocity to average speed is:

(A) Always less than one

(B) Always equal to one

(C) Always more than one

(D) Equal to or less than one

9. The distance travelled by a body is directly proportional to the time taken. Its speed:

(A) Increases

(B) Decreases

(C) Becomes zero

(D) Remains constant

10. Which of the following can be zero, when a particle is in motion for some time?

(A) Distance

(B) Displacement

(C) Speed

(D) None of them

11. If the velocity changes with time. It means that the object may be:

(A) Accelerating

(B) Decelerating

(C) Accelerating and decelerating both

(D) Moving in a uniform velocity

12. A stone falls from a balloon that is descending at a uniform rate of 12m/s. The displacement of the stone from the point of release after 10 sec is:

(A) 490 m

(B) 510 m

(C) 610 m

(D) 725 m

13. A body is dropped from a height of 19.6 m. After falling through 4.9 m, the gravity ceases to act, and the body falls down with constant speed. The ratio of times of fall with gravity and without gravity:

(A) 1: 2

(B) 2: 3

(C) 2: 1

(D) 3: 2

14. Water drops fall from a tap at a height h above the ground. The drops fall at regular intervals of time. When the first drop touches the ground, fifth drop is ready to fall. Then separation between the second and third drops as:

(A) 5h/16

(B) 6h/16

(C) 4h/16

(D) h/5

15. A boy sees a ball going up and then back down through a window of 2.45 m high. If the total time the ball is in sight for up and down motion is 1 sec. The height above the window that the ball rises is:

(A) 0.98 m

(B) 0.49 m

(C) 0.245 m

(D) 0.306m

16. A body projected vertically up with velocity $\sqrt{4gh}$ strikes a glass plate at a height 'h' above the ground from which it is projected. After breaking the glass, it loses half of its velocity. Then it can go up to a further height of:

(A) h (B) h/2

(C) h/4 (D) h/8

17. A ball thrown up vertically with an initial speed of 20ms^{-1}. When it has reached 3/4 of the maximum height, speed of that ball is: (g = 10 ms^{-2})

(A) 10 m/s

(B) 5 m/s

(C) 15 m/s

(D) 20 m/s

18. A food packet is released from a helicopter which is rising at 4 ms–1. Then velocity of the packet after 2 seconds is:

(A) 15.6 ms^{-1} up

(B) 15.6 ms^{-1} down

(C) 17.6 ms^{-1} up

(D) 17.6 ms^{-1} down

19. A balloon rises from rest with a constant acceleration $\frac{g}{8}$. A stone is released from it when it has risen to a height h. The time taken by the stone to reach the ground is:

(A) $\sqrt{\frac{h}{g}}$ (B) $\sqrt{\frac{2h}{g}}$

(C) $2\sqrt{\frac{h}{g}}$ (D) $4\sqrt{\frac{h}{g}}$

20. From a balloon moving down at 10 ms^{-1} an object is dropped when it is at an altitude of 40m. The object strikes the ground with a velocity equal to (g = 10 ms^{-2})

(A) 20 ms^{-1}

(B) $10\sqrt{7}$ ms^{-1}

(C) 30 ms^{-1}

(D) 40 ms^{-1}

21. A body A is projected upwards with a velocity of 98m/s. The second body B is projected upwards with the same initial velocity but after 4 sec. Both the bodies will meet after:

(A) 6 sec

(B) 8 sec

(C) 10 sec

(D) 12 sec

22. An iron ball and a wooden ball of the same radius are released from the same height in vacuum. They take the same time to reach the ground. The reason for this is:

(A) Acceleration due to gravity in vacuum is same irrespective of the size and mass of the body

(B) Acceleration due to gravity in vacuum depends upon the mass of the body

(C) There is no acceleration due to gravity in vacuum

(D) In vacuum there is a resistance offered to the motion of the body and this resistance depends upon the mass of the body

23. A body, thrown upwards with some velocity reaches the maximum height of 50 m. Another body with double the mass thrown up with double the initial velocity will reach a maximum height of:

[BHU 2004]

(A) 100 m (B) 200 m
(C) 300 m (D) 400 m

Multi Correct Question (M.C.Q)

24. A particle is projected vertically upward with velocity u from a point, when it returns to point of projection:

(A) Its average speed is u/2
(B) Its average velocity is zero
(C) Its displacement is zero
(D) Its velocity speed is u

25. A particle is projected vertically upwards in vacuum with a speed v.

(A) The time taken to rise to half its maximum height is half the time taken to reach its maximum height

(B) The time taken to rise to three–fourth of its maximum height is half the time taken to reach its maximum height

(C) When it rises to half its maximum height, its speed becomes $v/\sqrt{2}$

(D) When it rises to half its maximum height, its speed becomes v/2

26. A stone is released from the top of a tall cliff. After n seconds another stone is thrown vertically downwards with a velocity u m/s. At what distance from the top of cliff the second stone will overtake the first and at what time?

(A) $t = \dfrac{(gn/2 - u)n}{gn - u}$

(B) $h = \dfrac{1}{2}g\left(\dfrac{(gn/2 - u)n}{gn - u}\right)^2$

(C) t = gn – u

(D) $h = \dfrac{1}{2}g(gn - u)^2$

Comprehension Passage (C.P.T)

A ball thrown up from the ground reaches a maximum height of 20m.

27. Find its initial velocity:

(A) 19.8 m/s

(B) 18.9 m/s

(C) 14.6 m/s

(D) 15.9 m/s

28. Find its velocity just before hitting the ground:

(A) –18.9 m/s

(B) 19.8 m/s

(C) 18.9 m/s

(D) –19.8 m/s

29. Find its displacement between 0.5 s and 2.5 s.

(A) 10.2 m

(B) 20.1 m

(C) 30.4 m

(D) 40.5 m

Matrix Matching Type (M.M.T.)

For a freely falling body, match the following.

Column - I

30. Ratio of the distance travelled in the 1st second, 2nd second, 3rd second...... is

31. Ratio of the distance travelled in 1 second, 2 second, 3 second, ... is

32. Ratio of the velocities acquired at the end of 1 second, 2 second, 3 second ..... is

33. Ratio of the time taken to travel 1st metre, 2nd metre, 3rd metre, ... is

Column - II

(A) $1^2: 2^2: 3^2:$ ....

(B) 1: 2: 3:....

(C) $1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2})$

(D) 1: 3: 5: ......

(E) 1: 4: 9: ......

Assertion Reason Type (A.R.T.)

(A) Both assertion & Reason are true, Reason is the correct explanation of assertion

(B) Both assertion & Reason are true, Reason is not the correct explanation of assertion

(C) Assertion is true but reason is false

(D) Assertion is false but reason is true

34. Assertion: A vector has both magnitude and direction.

Reason: The magnitude of a vector is a scalar.

35. Assertion: Equation of motion are applicable to freely falling bodies.

Reason: Acceleration due to gravity is constant up to certain height.

Integer Type Question (I.T.Q.)

36. An object is thrown upward with a speed of 20 m/s. How long will it take to reach the highest point? ($g = 10\ m/s^2$)

37. A body falls freely for 4 seconds. What is its final velocity? ($g = 10\ m/s^2$)

Previous Contest Question (P.C.Q.)

38. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of: [MNR 1998]

(A) 3s

(B) 5s

(C) 7s

(D) 9s

39. A body is thrown vertically upwards. If air resistance is to be taken into account, then the time during which the body rises is:

[RPET 2000; KCET 2001; DPMT 2001]

(A) Equal to the time of fall

(B) Less than the time of fall

(C) Greater than the time of fall

(D) Twice the time of fall

40. A ball P is dropped vertically, and another ball Q is thrown horizontally with the same velocities from the same height and at the same time. If air resistance is neglected, then: [MNR 1986; BHU 1994]

(A) Ball P reaches the ground first

(B) Ball Q reaches the ground first

(C) Both reach the ground at the same time

(D) The respective masses of the two balls will decide the time

41. A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is:
[CPMT 1983; Kerala PMT 2002]

(A) 4.9m

(B) 9.8m

(C) 19.6m

(D) 24.5m

42. An object is projected upwards with a velocity of 100 m/s. It will strike the ground after (approximately):

[NCERT 1981; AFMC 1995]

(A) 10 sec

(B) 20 sec

(C) 15 sec

(D) 5 sec

43. A body falls from rest, its velocity at the end of first second is: (g=32ft/sec)
[AFMC 1980]

(A) 16ft/sec

(B) 32ft/sec

(C) 64ft/sec

(D) 24ft/sec

44. A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity 3u. The height of the tower is:

[EAMCET 1983; RPET 2003]

(A) $3u^2/g$

(B) $4u^2/g$

(C) $6u^2/g$

(D) $9u^2/g$

45. Two stones of different masses are dropped simultaneously from the top of a building:

[EAMCET 1978]

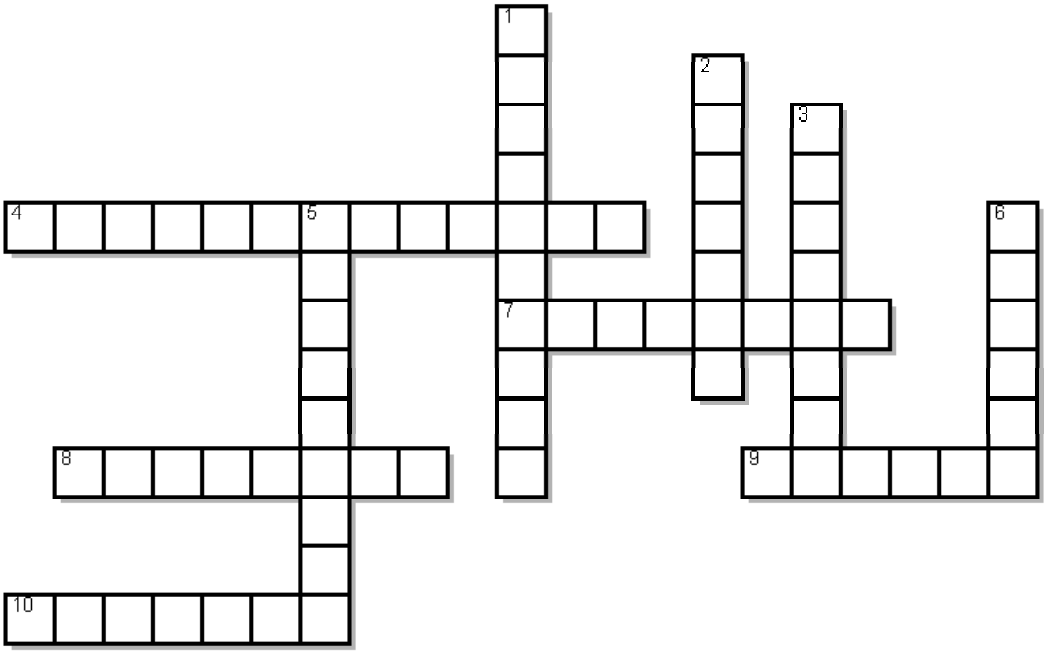
(A) Smaller stone hit the ground earlier

(B) Larger stone hit the ground earlier

(C) Both stones reach the ground simultaneously

(D) Which of the stones reach the ground earlier depends on the composition of the stone

Puzzle Time



Across (→)

4. If two vectors are _________ to each other, then their dot product is zero
7. _________ law of vectors is known as Lami's theorem
8. Objects in motion under the influence of gravity are said to be in _________
9. The total time during which the body thrown upwards remains in air is called time of _________
10. Physical quantities having only magnitude are called _________

Down (↓)

1. The process of splitting a vector is called _________ of a vector
2. The time taken by the freely falling body to reach the ground is called _________
3. The cross product of two _________ vectors is zero
5. All freely falling objects accelerate _________
6. The time taken by a projected body to reach the maximum height is called time of _________

NOTES