Chapter

1

QUADRATIC EXPRESSIONS

Sridhara Acharya was an Indian mathematician who lived around the 10th century. He wrote several treatises on the two major fields of Indian mathematics, pati-ganita and bija-ganita.

He is famous for his writings on the practical applications of Algebra, and he was one who gave a formula for solving quadratic equations for the first time.

CONCEPT MAP

Quadratic Equation

Quadratic Equation (degree 2)

General Form ($ax^2 + bx + c = 0$)

Solutions

METHOD 1

Factorisation

Solutions of a Quadratic Equation by Factorisation
If we can factorise $ax^2 + bx + c$, $a \neq 0$, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

METHOD 2

Completing Square

Solutions of a Quadratic Equation by completing the Square
A quadratic equation can also be solved by the method of completing the square

METHOD 3

Quadratic Formula

Nature of Roots

Discriminant (D)

- $D > 0$ Real distinct roots
- D-Equal roots
- $D < 0$ No real roots

CONCEPT 1.1

Quadratic Expression:

If $a \neq 0$, b, c are real (or) complex numbers then ax^2+bx+c is called a quadratic expression in x

Ex: $x^2-7x+12$, $x^2+8x+12$, $x^2-(1+2i)x+38$.

- A complex number 'α' is said to be a 'zero' of quadratic expression ax^2+bx+c, if $a\alpha^2+b\alpha+c=0$

Ex: 3 is a zero of x^2-5x+6

Quadratic Equation: **If $a \neq 0$, b, c are real (or) complex numbers then $ax^2+bx+c=0$ is called a quadratic equation in x.**

Ex: $x^2+5x+6=0, x^2+x+1=0$

Identity:

A relation which is true for every value of the variable is called an identity

Quadratic Identity:

$ax^2+bx+c=0$ will be an identity (or can have more than two solutions) if a = 0, b = 0 and c = 0.

Root of a Quadratic Equation:

If $a\alpha^2+b\alpha+c=0$ then α is a root or solution of the quadratic equation $ax^2+bx+c=0$.

- The roots of the quadratic equation $ax^2+bx+c=0$ are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and its discriminant is $\Delta=b^2-4ac$.

Ex: The roots of $3x^2-5x-12=0$ are

Sol: $x=\frac{5\pm\sqrt{25-4(3)(-12)}}{2\times 3}$ i.e. $x=-\frac{4}{3},3$ or (using factorization method)

$$3x^2-9x+4x-12=0 \Rightarrow 3x(x-3)+4(x-3)=0 \Rightarrow (3x+4)(x-3)=0$$

$$\Rightarrow x=-\frac{4}{3},3$$

- If α and β are the roots of $ax^2+bx+c=0$ then ,

  i) $\alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a}$ ii) $|\alpha-\beta|=\frac{\sqrt{b^2-4ac}}{|a|}$

iii) $\alpha^2+\beta^2=\dfrac{b^2-2ac}{a^2}$ iv) $\alpha^3+\beta^3=\dfrac{3abc-b^3}{a^3}$

- If α and β are the roots of $ax^2+bx+c=0$ then the quadratic expression $ax^2+bx+c=a(x-\alpha)(x-\beta)$
- The quadratic equation whose roots are α,β is given by $x^2-(\alpha+\beta)x+\alpha\beta=0$

Note:

Quadratic equation cannot have more than two roots.

Ex: The quadratic equation whose roots are $1+\sqrt{2},1-\sqrt{2}$ is

Sol: $x^2-\left(1+\sqrt{2}+1-\sqrt{2}\right)x+\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)=0$ i.e. $x^2-2x+(-1)=0$

$x^2-2x-1=0$ (or) $x=1+\sqrt{2}$

$x-1=\sqrt{2}$ $(x-1)^2=2\Rightarrow x^2-2x-1=0$

Ex: If $i^2=-1$, then the quadratic equation whose roots are $\dfrac{3\pm i\sqrt{5}}{2}$ is

Sol: $x^2-\left(\dfrac{3+i\sqrt{5}}{2}+\dfrac{3-i\sqrt{5}}{2}\right)x+\left(\dfrac{3+i\sqrt{5}}{2}.\dfrac{3-i\sqrt{5}}{2}\right)=0$

$x^2-3x+\dfrac{7}{2}=0\Rightarrow 2x^2-6x+7=0$ or $x=\dfrac{3+i\sqrt{5}}{2}\Rightarrow(2x-3)^2=-5$

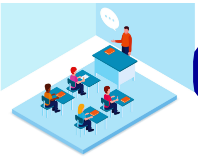
$\Rightarrow 2x^2-6x+7=0$

Ex: If α,β are the roots of $ax^2+bx+c=0$ and $c\neq 0$ then the value of $\dfrac{1}{(a\alpha+b)^2}+\dfrac{1}{(a\beta+b)^2}$ in terms of a, b, c is

Sol: $\because \alpha$ is a root of $ax^2+bx+c=0$

$\Rightarrow a\alpha^2+b\alpha=-c\Rightarrow a\alpha+b=\dfrac{-c}{\alpha}$, similarly $a\beta+b=\dfrac{-c}{\beta}$

$\therefore \dfrac{1}{(a\alpha+b)^2}+\dfrac{1}{(a\beta+b)^2}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{c^2}=\dfrac{b^2-2ac}{a^2c^2}$



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.1

1. A quadratic expression is an expression of the form:

(A) $ax + b$

(B) $ax^2 + bx + c$, where $a \neq 0$

(C) $ax^3 + bx^2 + c$

(D) $ax^2 + b$

2. The degree of a quadratic expression is:

(A) 1 (B) 2

(C) 3 (D) 0

3. If 3 is a zero of $2x^2 - 5x + 6$, then which of the following is true?

(A) $2(3)^2 - 5(3) + 6 = 0$

(B) $3(2)^2 - 5(2) + 6 = 0$

(C) $2x^2 - 5x + 6 = 0$

(D) $3(2)^2 + 5(3) + 6 = 0$

4. Which of the following is not a quadratic expression?

(A) $x^2 + 5x + 6$ (B) $3x^2 - 7$

(C) $2x^3 + 3x^2 + 1$ (D) $4x^2 + 9$

5. The number of zeros (roots) of a quadratic expression is:

(A) One (B) Two

(C) Three (D) Infinite

6. The quadratic equation corresponding to the expression $2x^2 + 3x - 5$ is:

(A) $2x^2 + 3x - 5 = 0$

(B) $2x^2 + 3x + 5 = 0$

(C) $2x^2 - 3x - 5 = 0$

(D) $2x^2 - 3x + 5 = 0$

7. A quadratic identity can have more than two solutions only if:

(A) $a = 0$

(B) $b = c = 0$

(C) $a = 0, b = 0, c = 0$

(D) $a = 0, b = c \neq 0$

8. Which of the following statements is true?

(A) A quadratic equation can have more than two roots.

(B) A quadratic equation can have only one root.

(C) A quadratic equation can have at most two roots.

(D) A quadratic equation can have three roots.

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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CONCEPT 1.2

Nature of the Roots of the Equation

$ax^2 + bx + c = 0$:

- If a, b, c are real and

  i) $\Delta > 0$, then the roots are real and distinct

  ii) $\Delta = 0$, then the roots are real and equal

  iii) $\Delta < 0$, then the roots are non- real conjugate complex numbers i.e., $\alpha \pm i\beta$

- If a, b, c are rational and

i) $\Delta > 0$ and is a perfect square then the roots are rational and distinct.

ii) $\Delta > 0$ and is not a perfect square then the roots are conjugate surds i.e., $\alpha \pm \sqrt{\beta}$. $(\beta \neq 0)$

iii) $\Delta = 0$, then the roots are equal & rational

iv) $\Delta < 0$, then the roots are non-real conjugate complex numbers. i.e., $\alpha \pm i\ \beta$

Ex: The nature of the roots of $x^2 + x - 1 = 0$ is

Sol: $\Delta = (1)^2 - 4(1)(-1) = 5$ is not a perfect square.

$\Rightarrow$ roots are irrational and more over conjugate

Ex: The nature of the roots of $2x^2 + x + 6 = 0$ is

Sol: $\Delta = 1 - 4(2)(6) = -47 < 0$

$\Rightarrow$ roots are imaginary more over conjugate complex numbers

Ex: If the roots of the equation $x^2 - 15 - m(2x - 8) = 0$ are equal then the value of m is

Sol: $x^2 - 15 - m(2x - 8) = 0$

$x^2 - 2mx + 8m - 15 = 0$

$\Delta = 0 \Rightarrow (-2m)^2 - 4 \times 1 \times (8m - 15) = 0$

$m^2 - 8m + 15 = 0$

$(m-3)(m-5) = 0 \quad m = 3;\ m = 5$

Ex: If a, b, c are real, the roots of $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are real and equal, then the expression in which a, b, c lies is

Sol: $a^2x^2 + b^2x^2 - 2abx - 2bcx + b^2 + c^2 = 0$

$a^2x^2 - 2abx + b^2 + b^2x^2 - 2bcx + c^2 = 0$

$(ax - b)^2 + (bx - c)^2 = 0$

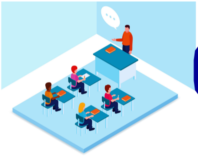
$ax - b = 0$; $bx - c = 0$; $x = \frac{b}{a}$; $x = \frac{c}{b}$

$\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$, $\therefore a, b, c$ are in G.P

Nature of the roots of two Quadratic Equations:

If Δ_1 and Δ_2 are the discriminants of two quadratic equations $P(x) = 0$ and $Q(x) = 0$, such that

- $\Delta_1 + \Delta_2 \geq 0$ then there will be at least two real roots for the equation $P(x) = 0$ or $Q(x) = 0$.
- If $\Delta_1 + \Delta_2 < 0$, then there will be at least two imaginary roots for the equation $P(x) = 0$ or $Q(x) = 0$.
- If $\Delta_1 . \Delta_2 < 0$, then the equation $P(x).Q(x) = 0$ will have two real roots and two imaginary roots.
- If $\Delta_1 . \Delta_2 > 0$, then the equation $P(x).Q(x) = 0$ has either four real roots or no real roots.
- If $\Delta_1 . \Delta_2 = 0$ such that $\Delta_1 > 0$ and $\Delta_2 = 0$ or $\Delta_1 = 0$ and $\Delta_2 > 0$ then the equation $P(x).Q(x) = 0$ will have two equal roots and two distinct roots.
- If $\Delta_1 . \Delta_2 = 0$ where $\Delta_1 < 0$ and $\Delta_2 = 0$ or $\Delta_1 = 0$ and $\Delta_2 < 0$ then the equation $P(x).Q(x) = 0$ will have two equal real roots.
- If $\Delta_1 \Delta_2 = 0$ such that $\Delta_1 = 0$ and $\Delta_2 = 0$ then the equation $P(x)Q(x) = 0$ will have two pairs of equal roots .



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.2

1. **The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by:**
   (A) $b^2 + 4ac$
   (B) $b^2 - 4ac$
   (C) $b^2 / 4ac$
   (D) $2b - 4ac$

2. **If the discriminant $(\Delta) > 0$, then the quadratic equation has:**
   (A) Real and equal roots
   (B) Real and distinct roots
   (C) No real roots
   (D) Complex roots

3. **If the discriminant $(\Delta) = 0$, then the quadratic equation has:**
   (A) Real and distinct roots
   (B) Real and equal roots
   (C) Imaginary roots
   (D) Surd roots

4. **If Ä < 0, then the roots of the quadratic equation are:**
   (A) Real and distinct
   (B) Real and equal
   (C) Non-real complex conjugates
   (D) Rational numbers

5. **The roots of $2x^2 + x + 3 = 0$ are:**
   (A) Real and distinct
   (B) Real and equal
   (C) Non-real complex
   (D) Rational

6. **If $\Delta > 0$ but not a perfect square, then the roots of the equation are:**
   (A) Rational and distinct
   (B) Real and distinct but irrational
   (C) Complex
   (D) Equal

7. **For what values of b will the equation $x^2 + bx + 1 = 0$ have equal roots?**
   (A) $b = 1$
   (B) $b = -2$
   (C) $b = 2$ or -2
   (D) $b = \pm 2$

8. **The nature of roots of $x^2+x+1=0$ is:**
   (A) Real and equal
   (B) Real and distinct
   (C) Non-real complex
   (D) Rational

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CONCEPT 1.3

Common root:

The equations $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ where $a_1b_2-a_2b_1\neq 0$, $a_1\,a_2\neq 0$ have a common root if and only if

$(c_1a_2-c_2a_1)^2=(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)$ and the common root is

$\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$.(or) $\frac{b_1c_2-b_2c_1}{c_1a_2-c_2a_1}$

Ex: If $x^2+4ax+3=0$ and $2x^2+3ax-9=0$ have a common root then the value of a is

Sol: Condition for the common roots is

$(c_1a_2-c_2a_1)^2=(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)\Rightarrow(6+9)^2=(3a-8a)(-36a-9a)\Rightarrow a=\pm 1$

Note:

If $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ have the same roots then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Properties of roots of the quadratic equation:

$ax^2+bx+c=0$: If a and c are of the same sign then $\frac{c}{a}$ is +ve and hence roots have same signs

- If a and c are of the opposite signs then $\frac{c}{a}$ is -ve and hence roots have opposite sign
- If $a=c$ then the roots are reciprocal to each other.
- If both the roots are -ve then a,b,c will have the same sign
- If both roots are +ve then a,c will have the same sign different from the sign of b
- If a+b+c=0 then the roots are 1 and $\frac{c}{a}$
- If a+c=b then the roots are -1 and $\frac{-c}{a}$
- If the roots are in the ratio m: n then $(m+n)^2ac=mnb^2$ (or) $\frac{(m+n)^2}{mn}=\frac{b^2}{ac}$
- If one root is square of the other then $a^2c+c^2a=b(3ac-b^2)$
- If one root is equal to the n^{th} power of the other root

  then $(ac^n)^{\frac{1}{n+1}}+(a^nc)^{\frac{1}{n+1}}+b=0$.
- If roots differ by k then $b^2-4ac=a^2k^2$.

Ex: The roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ $(a \neq b \neq c \neq 0)$ are

Sol: $a(b-c)+b(c-a)+c(a-b)=0$

$\therefore$ the roots are $1, \dfrac{c(a-b)}{a(b-c)}$

Ex: If $p(q-r)x^2+q(r-p)x+r(p-q)=0$ $(p \neq q \neq r)$ has equal roots the value of $\dfrac{2}{q}$ in terms of p & q is

Sol: $p(q-r)+q(r-p)+r(p-q)=0$

$\therefore$ roots are $1, \dfrac{r(p-q)}{p(q-r)}$

$\because$ roots are equal $\Rightarrow$ $1=\dfrac{r(p-q)}{p(q-r)}$

$p(q-r)=r(p-q)$ $\Rightarrow pq-pr=rp-rq$

$pq+rq=2pr \Rightarrow (p+r)q=2pr = \dfrac{p+r}{pr}=\dfrac{2}{q}$

Ex: The roots of $ax^2+3bx+c=0$ if $3b=a+c$ are

Sol: $ax^2+3bx+c=0$ and $3b=a+c$

$ax^2+(a+c)x+c=0$; $ax^2+ax+cx+c=0$

$ax(x+1)+c(x+1)=0$; $(x+1)(ax+c)=0$

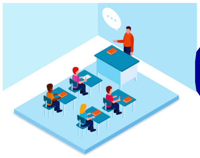
$x=-1$; $x=\dfrac{-c}{a}$

$\therefore$ roots are $-1, \dfrac{-c}{a}$

Ex: If one root of the equation $ax^2+bx+c=0$ is double the other, then the relation between a, b, c is

Sol: Condition is $\dfrac{(m+n)^2}{mn}=\dfrac{b^2}{ac}$ given $m:n=1:2$

$\therefore \dfrac{(1+2)^2}{1\times 2}=\dfrac{b^2}{ac} \Rightarrow 9ac=2b^2$



CLASSROOM DISCUSSION QUESTIONS

CDQ 1.3

1. Two quadratic equations have a common root if:

(A) Their coefficients are equal

(B) Their discriminants are equal

(C) They satisfy the common root condition

(D) They have the same constant term

2. If both roots of $ax^2 + bx + c = 0$ have the same sign, then:

(A) a and c have the same sign

(B) a and c have opposite signs

(C) a and b have same sign

(D) b and c have opposite signs

3. If a = c in $ax^2 + bx + c = 0$, then the roots are:

(A) Equal

(B) Reciprocals of each other

(C) Imaginary

(D) Opposite

4. If both roots of $ax^2 + bx + c = 0$ are negative, then:

(A) a, b, c have same sign

(B) a and c have opposite signs

(C) a and b have opposite signs

(D) b and c have opposite signs

5. If $a + b + c = 0$, then the roots of $ax^2 + bx + c = 0$ are:

(A) 1 and – 1

(B) 1 and a/c

(C) 1 and $-c/a$

(D) 1 and c/a

6. If $a + c = b$, then the roots of $ax^2 + bx + c = 0$ are:

(A) – 1 and $-c/a$

(B) –1 and c/a

(C) –1 and a/c

(D) 1 and c/a

7. If the roots are in the ratio m : n, then the condition is:

(A) $b^2 = a c$

(B) $b^2 = a c (m/n)$

(C) $b^2 = a c (m + n)^2 / (mn)$

(D) $b^2 = 4ac$

8. If one root is square of the other, then the relation between a, b, c is:

(A) $a^2 + c^2 = 2b^2$

(B) $a^2 + b^2 = c^2$

(C) $a^2 + c^2 + b^2 = 0$

(D) $a^2 + c^2 = b^2 + 3ac$

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CONCEPT 1.4

Transformed Equations:

Let α, β be the roots of $f(x) \equiv ax^2+bx+c=0$ then

S.No	New Roots	Corresponding Quadratic Equation
1	$-\alpha, -\beta$	$f(-x)=0$
2	$\frac{1}{\alpha}, \frac{1}{\beta}$	$f\left(\frac{1}{x}\right)=0$
3	$k\alpha, k\beta \ (k \neq 0)$	$f\left(\frac{x}{k}\right)=0$
4	$\frac{\alpha}{k}, \frac{\beta}{k} \ (k \neq 0)$	$f(kx)=0$
5	$\alpha+k, \beta+k$	$f(x-k)=0$
6	α^2, β^2	$f(\sqrt{x})=0$
7	α^3, β^3	$f(\sqrt[3]{x})=0$
8	$\frac{\alpha}{1+\alpha}, \frac{\beta}{1+\beta}$	$f\left(\frac{x}{1-x}\right)=0$

Ex: If α, β are the roots of $ax^2+bx+c=0$, then the equation whose roots are $2+\alpha$ and $2+\beta$ is

Sol: $f(x-2)=0 \Rightarrow a(x-2)^2+b(x-2)+c=0$

$\Rightarrow ax^2+x(b-4a)+4a-2b+c=0$

Ex: If each root of the equation $x^2+11x+13=0$ is diminished by 4, then the transformed equation is

Sol: $f(x+4)=0 \Rightarrow (x+4)^2+11(x+4)+13=0 \Rightarrow x^2+19x+73=0$

Ex: If α, β are the roots of $4x^2+7x+2=0$, then the equation whose roots are α^2, β^2 is

Sol: $f(x)=4x^2+7x+2=0$

The required equation is $f(\sqrt{x})=0$.

$\Rightarrow 4(\sqrt{x})^2+7\sqrt{x}+2=0 \Rightarrow 4x+2=-7\sqrt{x}$

$\Rightarrow (4x+2)^2=(-7\sqrt{x})^2$

$\Rightarrow 16x^2-33x+4=0$

- **Signs of 'a' and** ax^2+bx+c **:** If the equation $ax^2+bx+c=0$ has non real roots $(\Delta<0)$ then 'a' and ax^2+bx+c will have same sign $\forall x \in R$.
- If the equation $ax^2+bx+c=0$ has equal roots then 'a' and ax^2+bx+c will have same sign $\forall x \in R-\left\{\frac{-b}{2a}\right\}$.
- If the equation $ax^2+bx+c=0$ has real roots α and β $(\Delta>0, \alpha<\beta)$ then

(i) $\alpha<x<\beta \Leftrightarrow$ a and ax^2+bx+c will have opposite sign.

(ii) $x<\alpha$ or $x>\beta \Leftrightarrow$ a and ax^2+bx+c will have same sign.

Maximum And Minimum Values:

ax^2+bx+c; a, b, c $\in$ R:

(i) If a > 0 then $f(x)=ax^2+bx+c$ has absolute minimum at

$x=-\frac{b}{2a}$ and the minimum value is $\frac{4ac-b^2}{4a}$

(ii) If a < 0 then $f(x)=ax^2+bx+c$ has absolute maximum at

$x=-\frac{b}{2a}$ and the maximum value is $\frac{4ac-b^2}{4a}$

(iii) If $f(x)=\frac{ax^2+bx+c}{ax^2-bx+c}$ (or) $f(x)=\frac{ax^2-bx+c}{ax^2+bx+c}$, $(b^2-4ac<0)$, then the minimum and maximum values of $f(x)$ are given by $f\left(\pm\sqrt{\frac{c}{a}}\right)$

Ex: The maximum or minimum values of $2x+5-3x^2$ on R is

Sol: Given expression $2x+5-3x^2$, $a=-3<0, b=2, c=5$

$\Rightarrow$ given expression has maximum value

$\therefore$ maximum value $=\frac{4ac-b^2}{4a}=\frac{4(-3)(5)-4}{4(-3)}=\frac{16}{3}$

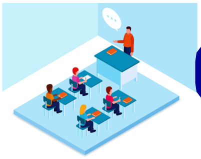
Ex: Find the range the expression $\frac{x^2-x+1}{x^2+x+1}$, where $x \in R$.

Sol: Here a=1,b=1,c=1, minimum value = f(1) = 1/3

maximum value $=f(-1)=3$

- If x is real then the maximum and minimum values of

$\frac{(a+x)(b+x)}{c+x}(x>-c, a>c, b>c)$ are $\left(\sqrt{a-c}-\sqrt{b-c}\right)^2$ and $\left(\sqrt{a-c}+\sqrt{b-c}\right)^2$



CLASSROOM DISCUSSION QUESTIONS — CDQ 1.4

1. **If a and b are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $-a$ and $-b$ is:**
(A) $f(x) = 0$
(B) $f(-x) = 0$
(C) $f(x + 1) = 0$
(D) $f(1/x) = 0$

2. **If a and b are roots of $f(x) = 0$, then the equation whose roots are $1/a$ and $1/b$ is:**
(A) $f(-x) = 0$
(B) $f(1/x) = 0$
(C) $x^2 f(1/x) = 0$
(D) $x f(1/x) = 0$

3. **If a and b are the roots of $f(x) = 0$, then the equation whose roots are ka and kb is:**
(A) $f(kx) = 0$
(B) $f(x/k) = 0$
(C) $k^2 f(x/k) = 0$
(D) $f(x) = 0$

4. **If a and b are roots of $f(x) = 0$, then the equation whose roots are $a + k$ and $b + k$ is:**
(A) $f(x + k) = 0$
(B) $f(x - k) = 0$
(C) $f(kx) = 0$
(D) $f(x/k) = 0$

5. **If each root of $x^2 + 11x + 13 = 0$ is diminished by 4, then the new equation is:**
(A) $x^2 + 19x - 76 = 0$
(B) $x^2 - 18x + 73 = 0$
(C) $x^2 + 19x + 73 = 0$
(D) $x^2 - 5x + 49 = 0$

6. **If a and b are roots of $x^2 + 2x + 1 = 0$, then the equation whose roots are $a - 3$ and $b - 3$ is:**
(A) $x^2 + 8x + 16 = 0$
(B) $x^2 + 4x + 4 = 0$
(C) $x^2 + 6x + 9 = 0$
(D) $x^2 + 2x + 1 = 0$

7. **If a and b are roots of $f(x) = 0$, then the equation whose roots are a^2 and b^2 is:**
(A) $f(x^2) = 0$
(B) $f\left(\sqrt{x}\right) = 0$
(C) $x^2 - (a^2 + b^2)x + (ab)^2 = 0$
(D) $x^2 - 2x + 1 = 0$

8. **If the roots of $x^2 - 2x + 3 = 0$ are increased by 1, then the new equation is:**
(A) $x^2 - 4x + 5 = 0$
(B) $x^2 - 3x + 4 = 0$
(C) $x^2 - 2x + 3 = 0$
(D) $x^2 - 4x + 6 = 0$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

C.D.F. CONCEPTS, DEFINITIONS AND FORMULAE

1. If $a, b, c \in R$ and $a \neq 0$, then the expression $ax^2 + bx + c$ is called a quadratic expression.
2. A real number a is said to be a zero of an expression ax^2+bx+c, if $aa^2 + ba + c = 0$
3. The value of a variable x for which satisfies the given equation $ax^2 + bx + c = 0$ is called **root** or **solution** of the equation.
4. The roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
5. If $ax^2 + bx + c = 0$ is a quadratic equation, then $b^2 - 4ac$ is called its discriminant.
6. If α, β are the roots of $ax^2+bx+c = 0$, then $ax^2+bx+c = a(x-\alpha)(x-\beta)$.
7. The quadratic equation having roots α, β is $(x-\alpha)(x-\beta) = 0$ or $x^2 - (\alpha+\beta)x + \alpha\beta = 0$.
8. If $f(x) = ax^2 + bx + c = 0$ is quadratic equation then whose roots are
   i) Greater by k than those of $f(x) = 0$ is $f(x - k) = 0$
   ii) Smaller by k than those of $f(x) = 0$ is $f(x + k) = 0$
9. If the roots of $ax^2+bx+c = 0$ are complex roots then for $x \in R$, ax^2+bx+c and a have the same sign.
10. If the roots of $ax^2+bx+c = 0$ are real and equal, then $\alpha = \frac{-b}{2a}$, for $\alpha \neq x \in R, ax^2+bx+c$ and 'a' have the same sign.
11. Let α, β be the real roots of $ax^2 + bx + c = 0$, $\alpha < \beta$, then
    i) $x \in R, \alpha < x < \beta \Rightarrow ax^2 + bx + c$ and a have opposite signs.
    ii) $x \in R$, $x < \alpha$ or $x > \beta \Rightarrow ax^2 + bx + c$ and 'a' have the same sign.
12. Let $f(x) = ax^2 + bx + c$ be a quadratic function.
    i) If $a > 0$ then f(x) has minimum value at $x = \frac{-b}{2a}$ and the minimum value $= \frac{4ac - b^2}{4a}$.
    ii) If $a < 0$ then f(x) has maximum value at $x = \frac{-b}{2a}$ and the maximum value $= \frac{4ac - b^2}{4a}$.

ADVANCED WORKSHEET

Single Correct Answer Type (S.C.A.T)

1. The nature of the roots of $x^2+x-1=0$ is:

(A) Not real

(B) Real and equal

(C) Irrational and conjugate to each other

(D) Rational and distinct

2. If the product of the roots of $5x^2-4x+2+m(4x^2-2x-1)=0$ is 3, then m = ______.

(A) 0

(B) – 1

(C) 1

(D) 2

3. If 3 + 4i and 3 – 4i are the roots of $x^2+px+q=0$, then (p, q) = ____.

(A) (6, 25)

(B) (6, 1)

(C) (–6, –7)

(D) (–6, 25)

4. Sum of the roots of a quadratic equation $2x^2+ax+5=0$ is $\frac{7}{2}$ then a = ____.

(A) 5 (B) –7

(C) 0 (D) 7

5. One root is six times the other for the equation $px^2-14x+8=0$, then p = ______.

(A) 1 (B) 2

(C) 3 (D) 4

6. The roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b) = 0$ are equal, then a, b, c are in:

(A) A.P (B) G.P

(C) H.P (D) A.G.P

7. The difference between the roots of $ax^2+bx+c=0$ is:

(A) $\sqrt{b^2-4ac}$

(B) $\frac{\sqrt{b^2-4ac}}{4a}$

(C) $\frac{\sqrt{b^2-4ac}}{a}$

(D) $\frac{-b}{a}$

8. If P is the ratio of the roots of an equation $ax^2+bx+c=0$, then $\frac{(p+1)^2}{P}$ = ______.

(A) $\frac{b}{ac}$ (B) $\frac{b^2}{ac}$

(C) $\frac{b}{a^2c^2}$ (D) $a^2b^2c^2$

9. If $x^2+bx+c=0$, $x^2+cx+b=0$ $(b \neq c)$ have a common root then $b + c =$_____.

(A) 0

(B) 1

(C) –1

(D) 2

10. If $\sqrt{x+1}-\sqrt{x-1}=1$, then x = __.

(A) $\frac{7}{11}$

(B) $\frac{2}{3}$

(C) $\frac{5}{4}$

(D) $-\left(\frac{3}{2}\right)$

11. The equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ has:

(A) No root

(B) One root

(C) Two roots

(D) Infinity

12. Find the value of λ for which the equation $\lambda x^2+2x+1=0$ has real and distinct roots:

(A) All real values of $\lambda < 5$

(B) All real values of $\lambda > 1$

(C) All real values of $\lambda \geq 1$

(D) All real values of $\lambda < 1$

13. Find the value of k if the equation $3x^2-2x+k=0$ and $6x^2-17x+12=0$ have a common root?

(A) $\frac{9}{2}$ or $\frac{13}{5}$

(B) $\frac{-8}{3}$ or $\frac{-15}{4}$

(C) $\frac{-7}{2}$ or $\frac{5}{9}$

(D) $\frac{-9}{4}$ or $\frac{-13}{3}$

14. Find the Quadratic equation with rational coefficients which has $2+\sqrt{3}$ and $2-\sqrt{3}$ as roots?

(A) $x^2-4x+1=0$

(B) $x^2+4x+1=0$

(C) $x^2-5x+4=0$

(D) $x^2-2\sqrt{3}+2\sqrt{3}x=0$

15. If each root of the equation $x^2+11x+13=0$ is diminished by 4, then the transformed equation is:

(A) $x^2+19x-76=0$

(B) $x^2-18x+73=0$

(C) $x^2+19x+73=0$

(D) $x^2-5x+49=0$

16. Find the equation whose roots are 3 more than the roots of the equation $x^2-2x-24=0$.

(A) $x^2-8x+9=0$

(B) $x^2-8x-9=0$

(C) $x^2+6x-8=0$

(D) $x^2-6x+8=0$

17. The equation whose roots are multiplied by 6 of those of $2x^2+3x+5=0$ is:

(A) $x^2-9x+90=0$

(B) $x^2-19x-90=0$

(C) $x^2+9x+90=0$

(D) $x^2+19x+19=0$

18. State the sign of x^2-x+1:

(A) Positive $\forall x \in R$

(B) Negative $\forall x \in R$

(C) Positive for $x<2$

(D) None

19. The roots of $ax^2+3bx+c=0$ if $3b=a+c$ are:

(A) $-1, \frac{c}{a}$ (B) $1, \frac{c}{a}$

(C) $-1, \frac{-c}{a}$ (D) None

20. If one root of the equation $ax^2+bx+c=0$ is double the other, then the relation between a,b,c is:

(A) $2b^2 = 19ac$

(B) $2b^2 = 5ac$

(C) $b^2 = 9ac$

(D) $2b^2 = 9ac$

21. If '1' is one of the roots of $ax^2+3x+5=0$ then the second root is:

(A) 5/8 (B) –5/8

(C) 5/16 (D) –5/16

22. The equation whose roots are $2\sqrt{3}-5$ and $-2\sqrt{3}-5$ is:

(A) $x^2+10x-13=0$

(B) $x^2-10x+13=0$

(C) $x^2+10x+13=0$

(D) $x^2-10x-13 = 0$

23. If α, β are roots of $ax^2+bx+c=0$ then the equation whose roots are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ is:

(A) $acx^2 - (b^2 - 2ac)x + ac = 0$

(B) $a^3x^2 + (b^3 - 3abc)x + c^3 = 0$

(C) $x^2 - 2qx + (q^2 - p^2) = 0$

(D) $x^2 + 2qx + (q^2 + p^2) = 0$

24. If α, β are the roots of the equation $ax^2+bx+c = 0$, then the quadratic equation whose roots are $\alpha+\beta, \alpha\beta$ is:

(A) $a^2x^2 + a(b - (c)x + bc = 0$

(B) $a^2x^2 + a(b - (c)x - bc = 0$

(C) $a^2x^2 + (b - (c)x + bc = 0$

(D) $a^2x^2 - (b + (c)x - bc = 0$

25. If α, β are the roots of $2x^2+x+3=0$, then the quadratic equation whose roots are $\alpha-1$, $\beta-1$ is:

(A) $2x^2 - x + 3 = 0$

(B) $3x^2 + x + 2 = 0$

(C) $2x^2 + 5x + 3 = 0$

(D) $2x^2 + 5x + 6 = 0$

26. The equation whose roots are greater by 1 than those of $2x^2-3x+1 = 0$:

(A) $3x^2 - 5x - 2 = 0$

(B) $2x^2 - 7x + 6 = 0$

(C) $2x^2 + 5x + 7 = 0$

(D) $3x^2 + 5x - 7 = 0$

27. If α, β are the roots of $3x^2+x+1=0$, then the quadratic equation whose roots are $3\alpha, 3\beta$ is:

(A) $27x^2 + 3x + 1 = 0$

(B) $3x^2 - x + 1 = 0$

(C) $x^2 + x + 3 = 0$

(D) None

28. The equation formed by decreasing each root of $ax^2+bx+c=0$ by 1 is $2x^2+8x+2=0$, then:

(A) $a = -b$

(B) $b = -c$

(C) $c = -a$

(D) $b = a + c$

29. If α, β are the roots of $x^2+5x-4=0$, then the equation whose roots are $\frac{\alpha+2}{3}, \frac{\beta+2}{3}$ is:

(A) $9x^2 + 3x + 10 = 0$

(B) $9x^2 - 3x - 10 = 0$

(C) $9x^2 + 3x - 10 = 0$

(D) $9x^2 - 3x + 10 = 0$

30. State the sign of x^2-5x+4.

(A) Positive if $x < 1$ or $x > 4$

(B) Positive if $x < -1$ or $x > 4$

(C) Positive if $x < 4$ or $x > 1$

(D) Positive if $x < -1$ or $x > 1$

31. For what values of x, x^2-5x+6 is negative:

(A) $2 < x < 3$

(B) $x < 2$ or $x > 3$

(C) $x = 2, 3$

(D) (A) & (B)

32. The maximum or minimum value of $3x-7+5x^2$ is:

(A) minimum = $-34/5$

(B) minimum = $-5/6$

(C) maximum = $11/20$

(D) None

33. The minimum value of $x^2-8x+17 \ \forall x \in R$ is:

(A) 17

(B) –1

(C) 1

(D) 2

34. If $2x-7-5x^2$ has maximum value at x = a, then the value of 'a' is:

(A) -1/5

(B) 1/5

(C) 34/5

(D) -34/5

35. The maximum value of $c+2bx-x^2$ is:

(A) b^2c

(B) $b^2 - c$

(C) $c - b^2$

(D) $b^2 + c$

36. The minimum value of the quadratic expression $x^2+2bx+c$ is:

(A) cb^2

(B) c^2b

(C) $c + b^2$

(D) $c - b^2$

37. The extreme value of $15+4x-3x^2$ is:

(A) $\frac{49}{3}$ (B) $-\frac{49}{3}$

(C) $\frac{47}{3}$ (D) $-\frac{47}{3}$

38. Find the maximum or minimum value of the Quadratic expression $3x^2+2x+7$:

(A) Maximum value 10/3

(B) Minimum value -10/3

(C) Maximum value -10

(D) Minimum value 20/3

39. The value of an quadratic equation $8a-a^2-15$ is maximum:

(A) 6

(B) 8

(C) 2

(D) 4

40. For what values of x, the expression $x^2-7x+10$ is negative

(A) $2<x<3$

(B) $2<x<5$

(C) $6<x<8$

(D) $-1<x<-2$

41. For what values of x, the expression $x^2-5x-14$ is positive:

(A) $x<-2$ or $x>7$

(B) $x<2$ or $x>1$

(C) $x<7$ and $x<-2$

(D) None

42. Find the solution for $\frac{x^2}{2}=5x-17$, if $i^2=-1$:

(A) $5\pm8i$

(B) $5\pm13i$

(C) $5\pm3i$

(D) $8\pm5i$

43. If each root of the equation $x^2+11x+13=0$ is diminished by 4, then the transformed equation is:

(A) $x^2+19x-76=0$

(B) $x^2-18x+73=0$

(C) $x^2+19x+73=0$

(D) $x^2-5x+49=0$

44. Find the Quadratic equation, the sum of whose roots is 1 and the sum of the squares of roots is 13.

(A) $x^2-x-6=0$

(B) $x^2+x-6=0$

(C) $x^2-x+6=0$

(D) $-x^2+x+6=0$

45. Quadratic equation whose roots are $\left(\frac{p-q}{p+q}\right), -\left(\frac{p+q}{p-q}\right)$ is:

(A) $(p^2+q^2)x^2+4pqx-(p^2-q^2)=0$

(B) $(p^2-q^2)x^2+4pqx-(p^2-q^2)=0$

(C) $(p^2+q^2)x^2-4pqx+(p^2-q^2)=0$

(D) None

Multi Corrrect Questions (M.C.Q.)

46. If $3^{2x^2-7x+7}=9$ then no. of real roots is:

(A) 1

(B) rational number

(C) 4

(D) composite number

47. If $x^2-2(1+3m)x+7(3+2m)=0$ has equal roots, then m = ____.

(A) 2 (B) -2

(C) $\frac{-10}{9}$ (D) $\frac{10}{9}$

48. If $x^2-hx-21 = 0$, $x^2-3hx+35 = 0$, have a common root then h = _____.

(A) – 2 (B) 2

(C) 4 (D) –4

49. α, β are roots of $f(x) = ax^2 + bx + c = 0$. Quadratic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ is:

(A) $f\left(\frac{1}{x}\right) = 0$

(B) $f(-x) = 0$

(C) $cx^2+bx+a=0$

(D) $ax^2 - bx + c = 0$

50. State the sign of $x^2 - 5x - 14$:

(A) Positive if $x < 2$ or $x > 7$

(B) Positive if $x < -2$ or $x > 7$

(C) Negative if $-2 < x < 7$

(D) Zero if $x = -2$ or 7

51. The expression $2x^2 + 4x + 7$ has minimum value m at $x = \alpha$, then

(A) $\alpha = 1$ (B) $\alpha = -1$

(C) $m = -5$ (D) $m = 5$

52. The expression $(x-a)(b-x)$ has

(A) maximum value at $x = \frac{a+b}{2}$

(B) minimum value at $x = \frac{a+b}{2}$

(C) maximum value $= \frac{(a-b)^2}{4}$

(D) minimum value $= \frac{(a-b)^2}{4}$

53. You want to frame a collage of pictures with a 9-ft strip of wood. Then:

(A) length = 2.25 ft

(B) width = 2.25 ft

(C) $l.w = 2.25$ will help you maximize the area

(D) Area = 2.0625 sq.ft

54. If both roots of $k(6x^2+3)+ rx +2x^2-1=0$ and $6k(2x^2+1)+ px +4x^2-2=0$ are common. Then:

(A) $\frac{p}{r} = 2$

(B) $p = 2r$

(C) $p-2r = 0$

(D) $p = r$

Comprehension Passage (C.P.T.)

PASSAGE - I

If α,β are the roots of $ax^2 + bx + c = 0$ then $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$.

55. If Sin θ, Cos θ are the roots of the equation $ax^2 + bx + c = 0$, then:

(A) $a^2-b^2+2ac=0$

(B) $a^2+b^2+2ac=0$

(C) $a-b+2ac=0$

(D) $a+b+2c = 0$

56. If α, β are the roots of $ax^2-2bx+c=0$, then $\alpha^3\beta^3+\alpha^2\beta^3+\alpha^3\beta^2$__.

(A) $\frac{c^2}{a^3}(c+2b)$ (B) $\frac{bc^3}{a^3}$

(C) $\frac{c^2}{a^3}(c-2b)$ (D) $\frac{c^2}{a^3}(2b-c)$

57. If α, β are the roots of $x^2-2x+4=0$, then $\alpha^4 + \beta^4$__.

(A) –8 (B) –16

(C) –4 (D) 8

PASSAGE - II

Let $f(x) = ax^2+bx+c$ be a quadratic function.

i) If a > 0 then f(x) has minimum value at

$x=\frac{-b}{2a}$ and the maximum value $=\frac{4ac-b^2}{4a}$

ii) If a < 0 then f(x) has maximum value at

$x=\frac{-b}{2a}$ and the maximum value $=\frac{4ac-b^2}{4a}$

58. $x^2-2x+10$ has minimum at x =________.

(A) 2 (B) –1

(C) 1 (D) –2

59. $3x-5x^2+12$ has maximum at x =3/10 and maximum value =__.

(A) $\frac{249}{20}$ (B) $-\frac{249}{5}$

(C) $\frac{3}{10}$ (D) $-\frac{249}{20}$

60. The maximum or minimum value of $\left(x-\frac{5}{3}\right)^2+\frac{7}{2}$ is:

(A) 1 (B) 7

(C) 7/5 (D) 7/2

Matrix Matching Type (M.M.T.)

I. α, β are roots of $f(x)=4x^2+x+1=0$. Quadratic equation whose roots are

Column-I

61. $-\alpha, -\beta$

62. $\frac{1}{\alpha}, \frac{1}{\beta}$

63. $\alpha+2, \beta+2$

64. $3\alpha, 3\beta$

Column-II

(A) $4x^2-15x+15 = 0$

(B) $4x^2-x+1 = 0$

(C) $4x^2 + 3x + 9 = 0$

(D) $16x^2 + 2x + 1 = 0$

(E) $x^2 + x + 4 = 0$

II. Column - I

65. The sign of $-6x^2+2x-3$

66. The sign of x^2+x+1 for $x \in R$ is

67. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2-(3a-1)x+2=0$ is twice as large as other is

68. $\tan 22^0$ and $\tan 23^0$ are roots of $x^2+ax+b=0$ then [if $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A \tan B}$]

Column - II

(A) $>0 \ \forall \ x \in R$

(B) $\frac{2}{3}$

(C) $<0 \ \forall \ x \in R$

(D) $a-b+1=0$

(E) $a+b+1=0$

Assertion Reason Type (A.R.T.)

(A) Both Assertion(A) and Reason(R) are correct and reason(R) is the correct explanation of assertion(A).

(B) Both Assertion(A) and Reason(R) are correct but reason(R) is not the correct explanation of assertion(A).

(C) Assertion(A) is correct but Reason(R) is incorrect.

(D) Assertion(A) is incorrect but Reason(R) is correct.

69. Assertion (A): The quadratic expression $f(x) = -3x^2+6x+1$ has a maximum value.

Reason (R): A quadratic expression ax^2+bx+c has a maximum value if $a < 0$.

70. Assertion (A): The maximum value of the quadratic expression $f(x) = -2x^2+8x-5$ is 3.

Reason (R): For $f(x)=ax^2+bx+c$, the maximum or minimum value is given by $f\left(-\frac{b}{2a}\right)$.

Integer Type Questions (I.T.Q.)

71. If α, β are the roots of $x^2+3x-2=0$ and $\frac{\alpha+1}{\beta}+\frac{\beta+1}{\alpha}=-k$ then k = ______.

72. If the difference of the squares of the roots of the equation $x^2-6x+q=0$ is 24, then the value of q is _____.

73. If '3' is root of $x^2+kx-24=0$ and it is also root of $x^2-kx+6=0$ then k = ______.

74. If α and β are the roots of $x^2-2x+4=0$ and the value of $\alpha^6+\beta^6$ is 2^m then m = ______ .

75. If the maximum value of $\frac{1}{4x^2+2x+1}$ is $\frac{4}{p}$ then p^2 = _____.

76. If x be real, then the maximum value of $5+4x-4x^2$ will be equal to: [MNR]

(A) 5 (B) 6

(C) 1 (D) 2

77. If x is real, then the maximum and minimum values of the expression $\frac{x^2-3x+4}{x^2+3x+4}$ will be:

[IIT]

(A) 2, 1 (B) 5, $\frac{1}{5}$

(C) 7, $\frac{1}{7}$ (D) None

Previous Question (P.Q.)

78. If x is real, then the value of $x^2-6x+13$ will not be less than:

[RPET]

(A) 4 (B) 6

(C) 7 (D) 8

79. If both the roots of $k(6x^2+3)+rx+2x^2-1=0$ and $6k(2x^2+1)+px+4x^2-2=0$ are common, then $2r-p$ is equal to:

[MNR]

(A) –1 (B) 0

(C) 1 (D) 2

80. If a root of the equations $x^2+px+q=0$ and $x^2+\alpha x+\beta=0$ is common, then its value will be (where $p \neq \alpha$ and $q \neq \beta$):

[iit ; rpet]

(A) $\frac{q-\beta}{\alpha-p}$

(B) $\frac{q-\beta}{\alpha+p}$

(C) $\frac{q-\beta}{\alpha-p}$ or $\frac{p\beta-\alpha q}{q-\beta}$

(D) $\frac{p\beta-\alpha q}{q-\beta}$

81. If x^2-3x+2 be a factor of x^4-px^2+q, then (p,q) = ____.

[IIT ; MP PET ; Pb. CET]

(A) (3, 4) (B) (4, 5)

(C) (4, 3) (D) (5, 4)

82. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then:

[iit ; mp pet]

(A) $a<2$ (B) $2\leq a\leq 3$

(C) $3\leq a\leq 4$ (D) $a>4$

83. If x be real, the least value of $x^2-6x+10$ is:

[Kurukshetra CEE]

(A) 1 (B) 2

(C) 3 (D) 10