Chapter

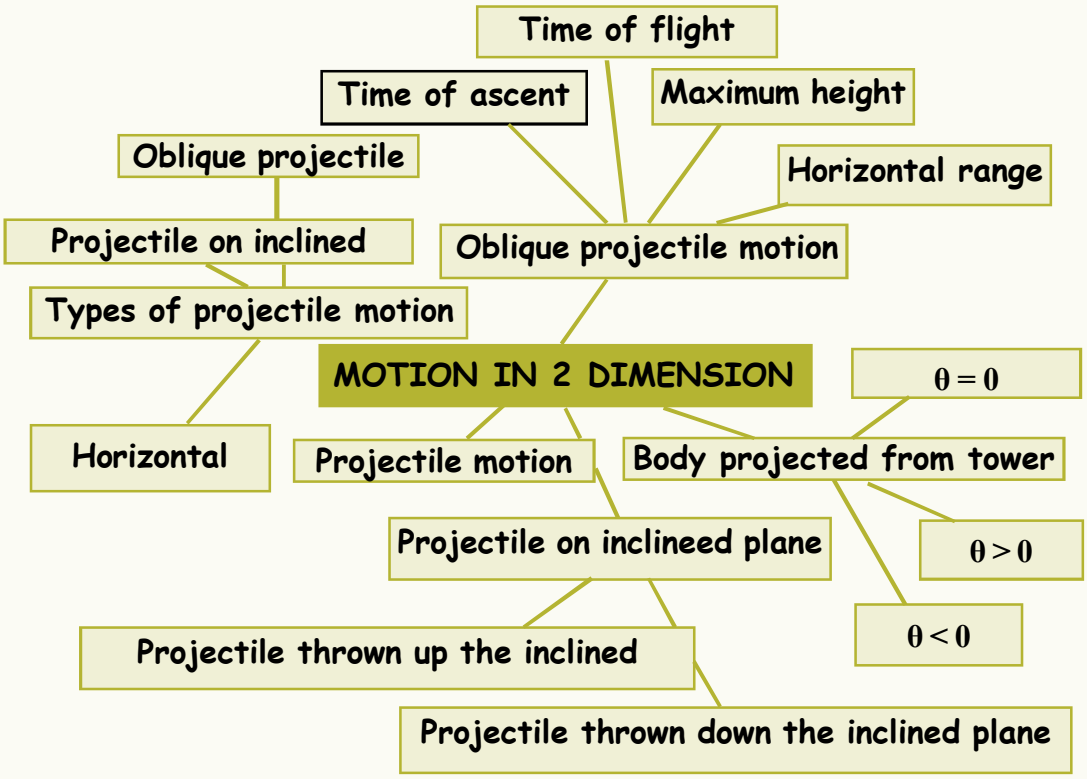
1

MOTION

Evangelista Torricelli - Italian mathematician and a student of Galileo Galilei, he invented mercury barometer, investigated theory of projectiles, improved telescope and invented a primitive microscope. Disproved that nature abhors and vacuum, presented Torricelli's theorem.

Perhaps his most notable achievement in the field of projectiles was to establish for the first time the idea of an envelope: projectiles sent out at the same angle and the same speed in all directions trace out parabolas which are all tangent to a common paraboloid.

CONCEPT MAP



CONCEPT 1

Mechanics :

Motion is caused by force. The branch of physics which deals with the effect of forces on objects is called mechanics.

Mechanics can be classified into two categories -

(i) Statics and

(ii) Dynamics

Statics:

It is the branch of mechanics which deals with objects at rest under the action of forces.

Dynamics:

It is the branch of mechanics which deals with objects under motion.

Dynamics can Again be Classified into Two Categories: -

(i) Kinematics and

(ii) Kinetics.

Kinematics:

It deals with the motion of objects without bothering about the cause of motion.

Kinetics:

It deals with the motion of objects considering the cause of their motion.

A Point Object:

An object is said to be a point object if its dimensions (i.e. length, breadth and thickness etc.) are negligible as compared to the distance travelled by it.

Basic Definitions:

Relative:

The word relative means comparing with something else. The terms like right and left; day and night; up and down are relative terms with respect to the position and point of view of the observer

Motion:

It is a relative quantity. A body is said to be in motion when its position is changing continuously with time relative to an observer.

Distance:

The length of the path traversed by an object in a given time of interval is called as "Distance". The distance is a scalar. Its SI unit is meter (m) and its C.G.S. unit is centimetre (cm). The dimensional formula of distance is $[M^0L^1T^0]$.

Displacement:

The shortest distance covered by an object in the specified direction is called "Displacement". It is a vector. Its SI unit is metre (m) and its C.G.S. unit is centimetre (cm). The dimensional formula of displacement is $[M^0L^1T^0]$.

Speed:

The distance travelled by a body per unit time is called speed.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

It's basic unit (or) SI unit is m/s and the CGS unit is cm/s.

Speed is a scalar quantity.

The dimensional formula of speed is $[M^0L^1T^{-1}]$.

The other units of speed are kmph (km/h).

Average speed:

The total distance covered by an object divided by total time taken is called "Average speed". It is a scalar.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Totla time}}$$

Velocity:

The rate of change of displacement is called "Velocity".

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

Its basic unit (or) SI unit is m/s and the CGS unit is cm/s.

Velocity is a vector quantity.

The dimensional formula of velocity is $[M^0L^1T^{-1}]$.

Average velocity:

The displacement of an object per unit time is called "Average velocity". It is a vector, and its direction is along the direction of displacement.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

Uniform motion:

The motion of a body is said to be in uniform when its velocity is constant.

Non - uniform motion:

The motion of an object is said to be nonuniform when its velocity is changing.

Change in velocity:

The change in velocity takes place in three ways.

i) Speed changes with direction remaining constant - Motion of a ball on a inclined plane

ii) Direction of motion changes while speed is constant - Uniform circular motion

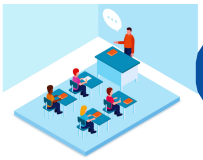
iii) Both direction and speed change continuously - Motion of an object thrown into air

Acceleration:

The rate of change of velocity is known as acceleration. It is a vector and acts along the direction of change of velocity. The SI unit of acceleration is m/s^2.

$$a = \frac{v-u}{t}$$

u - Initial velocity, v - Final velocity and t - time



CLASSROOM DISCUSSION QUESTIONS

CDQ 01

1. Which branch of mechanics deals with objects at rest under the action of forces?

(A) Dynamics (B) Kinematics
(C) Statics (D) Kinetics

2. What does kinematics deal with in terms of object motion?

(A) The cause of motion
(B) The effect of forces on objects
(C) The motion of objects without considering the cause
(D) The motion of objects considering the cause

3. What is the formula for average speed?

(A) Average speed = $\frac{\text{Distance}}{\text{Time}}$

(B) Average speed = $\frac{\text{Time}}{\text{Distance}}$

(C) Average speed=Distance × Time

(D) Average speed=Distance – Time

4. How is average velocity defined?

(A) As the distance covered by an object per unit time
(B) As the speed of an object in a specified direction
(C) As the rate of change of velocity
(D) As the displacement of an object per unit time

5. What is acceleration?

(A) The change in speed of an object
(B) The rate of change of velocity
(C) The distance traveled by an object per unit time
(D) The rate of change of speed of an object

6. Which of the following equations describes motion at uniform acceleration?

(A) $v = u+at$ (B) $S = ut+\frac{1}{2}at^2$

(C) $2aS = v^2-u^2$ (D) All

7. What does linear motion study with the help of displacement-time, velocity-time, and acceleration-time graphs?

(A) Circular motion
(B) Motion of an object on an inclined plane
(C) Motion of objects in a straight line
(D) Motion of objects in a plane

8. What is the SI unit of acceleration?

(A) m (B) s
(C) m/s (D) m/s^2

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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CONCEPT 2

Equations of motion:

The motion of an object moving at uniform acceleration can be described with the help of three equations, namely

$$v = u + at; \qquad S = ut + \frac{1}{2} at^2; \qquad 2aS = v^2 - u^2$$

where u is initial velocity of the object, which moves with uniform acceleration a for time t, v is its final velocity and s is the distance it travelled in time t.

Linear motion of a body can be studied with the help of the following graphs:

(i) Displacement - Time Graph or distance - time graph

(ii) Velocity - Time Graph

(iii) Acceleration - Time Graph

Acceleration Due to Gravity :

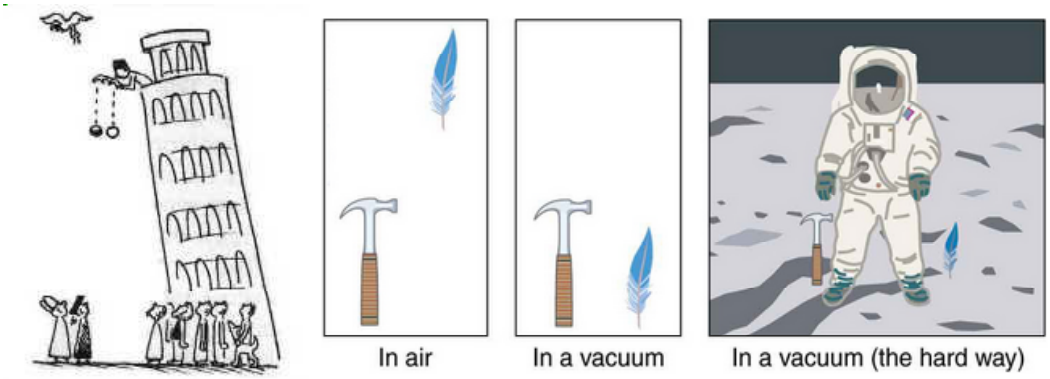
The uniform acceleration of a freely falling body towards the centre of earth due to earth's gravitational force is called "Acceleration due to gravity".

i) It is denoted by 'g'.

ii) Its value is constant for all bodies at a given place.

iii) It is independent of size, shape, material, constitution (hollow or solid), nature of the body.

iv) If air resistance is ignored, all the bodies as light as a feather to a heavy metal sphere, dropped simultaneously from the same height hit the floor at the same time because all the bodies have same acceleration due to gravity.



v) Its value changes from place to place on the surface of the earth.

vi) It has maximum value at the poles of the earth. The value is nearly 9.83 m/s^2.

vii) It has minimum value at the equator of the earth. The value is nearly 9.78 m/s^2.

viii) The average value of 'g' on the earth surface is 9.8 m/s^2.

ix) On the surface of moon, g = 1.67 m/s^2.

On the surface of sun, g = 27.4 m/s^2.

x) The acceleration due to gravity of a body is always directed downwards towards the centre of the earth, whether the body is projected upwards or downwards.

xi) When a body is falling towards the earth, its velocity increases and 'g' is positive.

xii) When a body is projected upwards, its velocity decreases and 'g' is negative.

xiii) The acceleration due to gravity at the centre of earth is zero.

Equations of Motion for Freely Falling Body:

Motion of all the dropped bodies falling towards the Earth when air resistance is ignored, is known as free fall.

For a freely falling body,

Initial velocity u = 0 m/s, acceleration a = + g

$v = u + gt$

Then the equations of motion become

1) $v = u + at \quad \Rightarrow \quad v = gt$

2) $S = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad S = \frac{1}{2}gt^2$

3) $v^2 - u^2 = 2aS \quad \Rightarrow \quad v^2 = 2gS$

4) $S_n = u + a\left(n - \frac{1}{2}\right) \quad \Rightarrow \quad S_n = g\left(n - \frac{1}{2}\right)$

Note:

1) For a freely falling body, the ratio of distances travelled in 1 second, 2 seconds, 3 seconds, 4 seconds . . . . . = 1: 4: 9: 16 . . . . . so on.

2) For a freely falling body, the ratio of distances travelled in successive seconds = 1: 3: 5: 9 . . . . . .so on.

3) In uniform accelerated motion, the distance travelled in every second increases by an amount equal to the magnitude of acceleration.

Equations of Motion for Vertically Projected upwards Body:

For a body projected vertically upwards, a = - g (since velocity, acceleration vectors are opposite).

1) $v = u + at \quad \Rightarrow \quad v = u - gt$

2) $S = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad S = ut - \frac{1}{2}gt^2$

3) $v^2 - u^2 = 2aS \quad \Rightarrow \quad v^2 - u^2 = 2gS$

4) $S_n = u + a\left(n - \frac{1}{2}\right) \quad \Rightarrow \quad S_n = u - g\left(n - \frac{1}{2}\right)$

CLASSROOM DISCUSSION QUESTIONS

CDQ 02

1. **What is the value of acceleration due to gravity at the surface of the Earth?**
   (A) 9.83 m/s² (B) 9.78 m/s²
   (C) 9.8 m/s² (D) 1.67 m/s²
2. **Where does the acceleration due to gravity have its maximum value on Earth?**
   (A) At the equator
   (B) At the poles
   (C) At the center
   (D) At the surface
3. **Which of the following statements about freely falling bodies is true?**
   (A) They always fall with different accelerations.
   (B) They have varying acceleration depending on their size.
   (C) They all have the same acceleration due to gravity.
   (D) Their acceleration depends on their material constitution.
4. **What is the acceleration of a freely falling body towards the Earth when air resistance is ignored?**
   (A) Zero (B) Negative
   (C) Positive (D) Constant
5. **What is the equation for the final velocity of a freely falling body with initial velocity 0 m/s?**
   (A) $v = gt$ (B) $v = u - gt$
   (C) $v^2 = 2gs$ (D) $v = u + gt$
6. **In the equations of motion for a freely falling body, what does 's' represent?**
   (A) Initial velocity
   (B) Time
   (C) Final velocity
   (D) Distance traveled
7. **What is the ratio of distances traveled in successive seconds for a freely falling body?**
   (A) 1 : 4 : 9 : 16
   (B) 1 : 3 : 5 : 9
   (C) 1 : 2 : 3 : 4
   (D) 1 : 1 : 1 : 1
8. **In which motion does the distance traveled in every second increase by an amount equal to the magnitude of acceleration?**
   (A) Uniform motion
   (B) Non-uniform motion
   (C) Freely falling motion
   (D) Linear motion

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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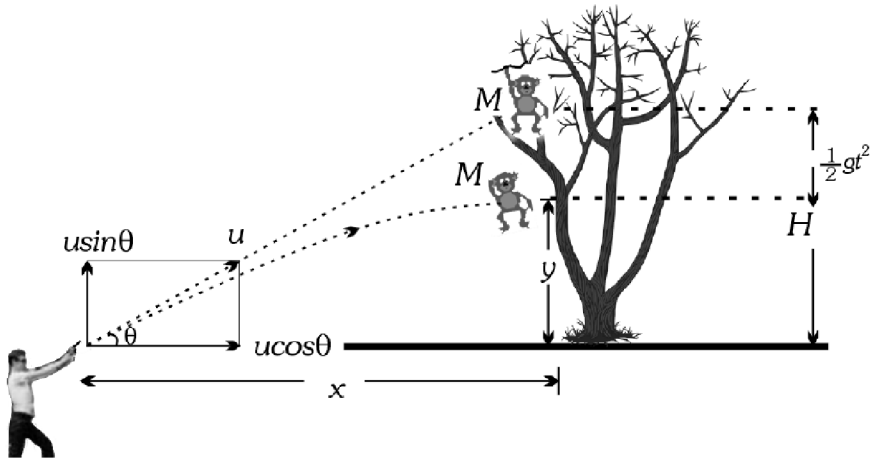
CONCEPT 3

Two Dimensional Motion:

The motion of an object is called two dimensional, if two of the three co-ordinates are required to specify the position of the object in space which changes w.r.t time. In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun etc. Two special cases of motion in two dimensions are

1. Projectile motion
2. Circular motion



A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, it will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path. The path of motion of a bullet will be parabolic and this motion of bullet is defined as projectile motion. If the force acting on a particle is oblique with initial velocity, then the motion of particle is called projectile motion.

Projectile:

Projectile is any body projected into the air at an angle other than 90° with the horizontal near the surface of the earth. The path followed by a projectile is called its trajectory. A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.

Example:

i) A bomb released from an aeroplane in level flight

ii) A bullet fired from a gun

iii) An arrow released from bow

iv) A Javelin thrown by an athlete

Assumptions of Projectile Motion:

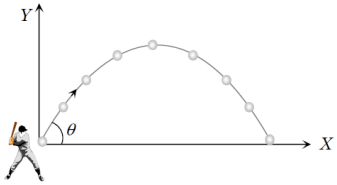
1. There is no resistance due to air.
2. The effect due to curvature of earth is negligible.
3. The effect due to rotation of earth is negligible.
4. For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

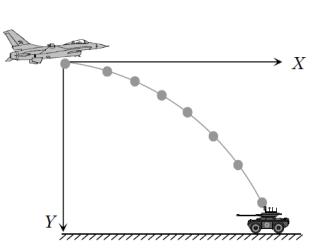
Principles of Physical Independence of Motions:

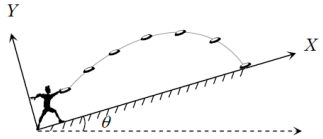
1. The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.
2. The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.
3. The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.
4. The horizontal motion is a uniform motion, and the vertical motion is a uniformly accelerate retarded motion.

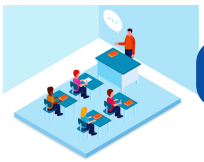
Types of Projectile Motion:

1. Oblique projectile motion
2. Horizontal projectile motion
3. Projectile motion on an inclined plane









CLASSROOM DISCUSSION QUESTIONS

CDQ 03

1. **What is the defining characteristic of two-dimensional motion?**
   (A) Motion along a straight line
   (B) Motion along a curved path
   (C) Motion in a plane
   (D) Motion in three dimensions
2. **Which of the following is an example of projectile motion?**
   (A) Car moving along a straight road
   (B) Boat sailing across a river
   (C) Bird flying in the sky
   (D) Arrow released from a bow
3. **What is the trajectory of a projectile?**
   (A) The force acting on the projectile
   (B) The path followed by the projectile
   (C) The initial velocity of the projectile
   (D) The distance traveled by the projectile
4. **Which assumption is made in the study of projectile motion?**
   (A) There is no air resistance
   (B) The Earth is flat
   (C) The effect of gravity is negligible
   (D) The motion occurs in one dimension
5. **What happens to the horizontal component of velocity during projectile motion?**
   (A) It increases continuously
   (B) It decreases continuously
   (C) It remains constant
   (D) It changes direction
6. **Which type of motion is a projectile's vertical motion during its flight?**
   (A) Uniform motion
   (B) Uniformly accelerated motion
   (C) Uniformly retarded motion
   (D) Non-uniform motion
7. **What is the trajectory of a projectile on an inclined plane called?**
   (A) Straight line
   (B) Circle
   (C) Parabola
   (D) Curve
8. **What is the path followed by a projectile if there is no air resistance and negligible effects from the Earth's curvature and rotation?**
   (A) Circular path
   (B) Parabolic path
   (C) Straight line
   (D) Elliptical path

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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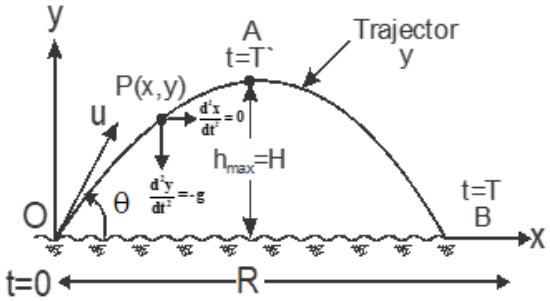
CONCEPT 4

Oblique Projectile:



In oblique projectile motion, horizontal component of velocity $(u\cos\theta)$, acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sin θ), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

Equation of The Oblique Projectile or Trajectory:

A projectile thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components i.e., u cos θ (component along X–axis) and u sin θ (component along Y–axis).

For horizontal motion $x = u\cos\theta \times t \Rightarrow t = \frac{x}{u\cos\theta}$ ..... (1)

For vertical motion $y = (u\sin\theta)t - \frac{1}{2}gt^2$ ..... (2) ($\because$ s = ut + 1/2 at^2)

From equations (i) and (ii) $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$ ..... (3)

This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola $y = ax - bx^2$

Note: Equation of oblique projectile also can be written as

$$y = x\tan\theta\left[1-\frac{x}{R}\right]$$ (Where R = horizontal range = $\frac{u^2\sin 2\theta}{g}$)

Time taken to Reach Maximum Height or Time of ascent:

Let this time be denoted t_a.

At the maximum height the vertical velocity is zero. i.e.,

$V_y = 0$

$U_y + a_y t = 0$ ($\because$ V = u + at)

$u \sin \theta - gt_a = 0 \qquad (\because a = -g \text{ and } u_y = u \sin\theta)$

$$\therefore \quad t_a = \frac{u \sin \theta}{g}$$

Time of Flight:

The total time spent by a projectile in air during the time of motion is called time of flight. On substituting y = 0 and t = T in equation (2) we can write

$$0 = u \sin \theta T - \frac{1}{2} g T^2 \Rightarrow T = \frac{2u \sin \theta}{g} \qquad \text{..... (4)}$$

Maximum Height:

As the projectile ascends (moves up), the vertical component of its velocity decreases. At maximum height of the vertical component of velocity becomes zero. That means the projectile moves horizontally at its highest position.

From the equation of motion $v^2 - u^2 = 2as$, the maximum height attained by a projectile can be written as $(0)^2 - (u \sin \theta)^2 = 2 \times -g \times H_m$

$$\Rightarrow \quad -u^2 \sin^2 \theta = -2gH_m$$

$$\Rightarrow \quad H_m = \frac{u^2 \sin^2 \theta}{2g} \qquad \text{..... (5)}$$

Note: When $\theta = 90°$, $H_{max} = \frac{u^2}{2g}$.

This is equal to the maximum height reached by a body projected vertically upwards.

Horizontal Range:

The horizontal distance travelled by the projectile while it touches the point on the same level of the point of projection is called horizontal range.

In the horizontal direction, velocity remains constant.

$$\therefore \quad \text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$u \cos \theta = \frac{R}{T} \qquad R = u \cos \theta . T \qquad \left(\because T = \frac{2u \sin \theta}{g}\right)$$

$$R = (u \cos \theta)\left(\frac{2u \sin \theta}{g}\right)$$

$$R = \frac{u^2 \sin 2\theta}{g} \qquad [\because 2 \sin \theta \cos \theta = \sin 2\theta] \qquad \text{..... (6)}$$

CLASSROOM DISCUSSION QUESTIONS

CDQ 04

1. **What remains constant in oblique projectile motion?**
   (A) Vertical component of velocity
   (B) Horizontal component of velocity
   (C) Acceleration
   (D) Mechanical energy

2. **What is the trajectory of a projectile in oblique projectile motion?**
   (A) Straight line (B) Parabola
   (C) Circle (D) Ellipse

3. **How is the velocity of a projectile resolved in oblique projectile motion?**
   (A) Into horizontal and vertical components
   (B) Into north and south components
   (C) Into forward and backward components
   (D) Into upward and downward components

4. **Which quantity is maximum at the point of projection in oblique projectile motion?**
   (A) Velocity
   (B) Kinetic energy
   (C) Momentum
   (D) Potential energy

5. **What is the time taken to reach maximum height in oblique projectile motion?**
   (A) $\frac{u\sin\theta}{g}$ (B) $\frac{u^2\sin\theta}{g}$
   (C) $\frac{u}{g}$ (D) $\frac{2u\sin\theta}{g}$

6. **What is the equation for the horizontal range R in oblique projectile motion?**
   (A) $R = \frac{u^2\sin 2\theta}{g}$
   (B) $R = \frac{u^2\sin\theta}{2g}$
   (C) $R = \frac{u^2\sin^2\theta}{g}$
   (D) $R = \frac{u^2\cos\theta}{g}$

7. **What happens to the vertical component of velocity at the maximum height in oblique projectile motion?**
   (A) It increases
   (B) It decreases
   (C) It remains constant
   (D) It becomes zero

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

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CONCEPT 5

Angle of Projection for the Maximum Range of the Projectile:

Range for an oblique projectile is given by, $R = \frac{u^2 \sin 2\theta}{g}$

The range R for a given velocity u is maximum if $\sin 2\theta = 1$

$$\Rightarrow \quad \sin 2\theta = \sin 90^\circ \Rightarrow 2\theta = 90^\circ \text{ or } \frac{\pi}{2} \Rightarrow \theta = 45^\circ \text{ or } \frac{\pi}{4}$$

Corresponding to this value of θ, the maximum range is

$$R_{maximum} = \frac{u^2}{g} \quad .....(7)$$

Relation between Range and Maximum Height:

We know that,

$$R = \frac{u^2 \sin 2\theta}{g} \text{ and } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\frac{H}{R} = \frac{\left[\frac{u^2 \sin^2 \theta}{2g}\right]}{\left[\frac{u^2 \sin 2\theta}{g}\right]} \Rightarrow \frac{H}{R} = \frac{u^2 \sin^2 \theta \times g}{2g \times u^2 \times 2 \sin\theta \cos\theta} = \frac{1}{4}\tan\theta$$

$$\text{Tan}\theta = \frac{4H}{R}$$

Sub Case:

Condition for range of projectile to be equal to maximum height attained by it

Tan θ = 4; $\theta = \tan^{-1}4 = 76^\circ$

Same Range at Different Angles of Projection:

Replace θ by $(90^\circ - \theta)$ in the formula of range we get

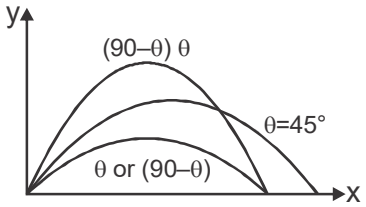
$$R = \frac{u^2 \sin 2(90^\circ - \theta)}{g} = \frac{u^2 \sin(180 - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g} = R$$

Thus, for a given velocity of projection, a projectile has the same range for angle of projection θ and $(90^\circ - \theta)$.

Note:

In the above case, the range of two projections is same but time of flights are different.

Relation between Time of Flights and Height of a Point in Case of Projectiles having Same Range:



Time of flight for angle of projection θ.

$$T_1 = \frac{2u\sin\theta}{g}$$

Time of the flight for angle of projection $(90° - \theta)$

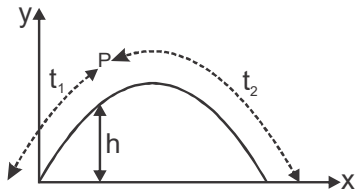
$$T_2 = \frac{2u\sin(90°-\theta)}{g} = \frac{2u\cos\theta}{g}$$

Multiplying T_1 and T_2 we get

$$T_1T_2 = \frac{2u\sin\theta}{g}\times\frac{2u\cos\theta}{g} \quad \text{or} \quad T_1T_2 = \frac{2}{g}\left(\frac{u^2\sin^2\theta}{g}\right)$$

$$T_1T_2 = \frac{2R}{g}; \qquad \frac{T_1}{T_2} = \text{Tan}\theta$$

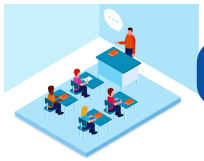
i) If t_1 is the time taken by projectile to reach a point P at height h and t_2 is the time taken from point P to ground level, then



$$t_1 + t_2 = T = \frac{2u\sin\theta}{g} \quad \text{or} \quad u\sin\theta = \frac{g(t_1+t_2)}{2}$$

The height of point P,

$$h = u\sin\theta t_1 - \frac{1}{2}gt_1^2 = \frac{g(t_1+t_2)}{2}t_1 - \frac{1}{2}gt_1^2 \quad \text{or} \quad h = \frac{1}{2}gt_1t_2$$



CLASSROOM DISCUSSION QUESTIONS

CDQ 05

1. **What angle of projection maximizes the range of a projectile?**

   (A) 45° (B) 60°

   (C) 75° (D) 90°

2. **What is the maximum range of a projectile in terms of its initial velocity u and acceleration due to gravity g?**

   (A) $\frac{u^2}{g}$ (B) $\frac{2u^2}{g}$

   (C) $\frac{u}{2g}$ (D) u^2g

3. **What is the condition for the range of a projectile to be equal to the maximum height attained by it?**

   (A) $\tan\theta = 4$ (B) $\sin\theta = 1$

   (C) $\cos\theta = 4$ (D) $\tan\theta = 1$

4. **At what angle of projection will a projectile have the same time of flight as another projectile projected at $90^0-\theta$?**

   (A) 45^0 (B) 30^0

   (C) 60^0 (D) 75^0

5. **What is the trajectory of a projectile in oblique projectile motion?**

   (A) Circle (B) Ellipse

   (C) Parabola (D) Hyperbola

6. **When does a projectile have the maximum vertical component of velocity?**

   (A) At the point of projection

   (B) At the maximum height

   (C) At the point of impact

   (D) At the midpoint of the trajectory

7. **How does the range of a projectile change with respect to the angle of projection?**

   (A) It decreases with increasing angle of projection

   (B) It increases with increasing angle of projection

   (C) It remains constant with respect to the angle of projection

   (D) It is not affected by the angle of projection

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

CONCEPT 6

Solved Examples:

EX 1: A body is projected with a velocity of 49 ms^{-1} at an angle 60^0 with the horizontal. What is the maximum height reached by the body? (Take g = 9.8 ms^{-2})

Sol: Maximum height reached $= \frac{u^2 \sin^2\theta}{2g}$

$$= \frac{49 \times 49\left(\frac{\sqrt{3}}{2}\right)^2}{2 \times 9.8} = \frac{5 \times 49 \times 3}{2 \times 4} = 91.875\, m$$

EX 2: Pankaj and Sudhir are playing with two different balls of masses m and 2m respectively. If Pankaj throws his ball vertically up and Sudhir at an angle q, both of them stay in our view for the same period. What is the ratio of heights attained by the two balls?

Sol: Time of flight for the ball thrown by Pankaj $T_1 = \frac{2u_1}{g}$

Time of flight for the ball thrown by Sudhir is given by

$$T_2 = \frac{2u_2 \sin(90^o - \theta)}{g} = \frac{2u_2 \cos\theta}{g}$$

According to problem $T_1 - T_2 \Rightarrow \frac{2u_1}{g} = \frac{2u_2 \cos\theta}{g} \Rightarrow u_1 = u_2 \cos\theta$

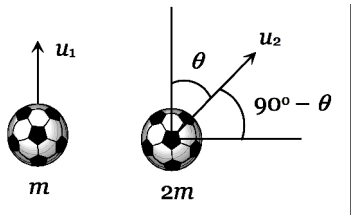
Height of the ball thrown by Pankaj $H_1 = \frac{u_1^2}{2g}$

Height of the ball thrown by Sudhir $H_2 = \frac{u_2^2 \sin^2(90^o - \theta)}{2g} = \frac{u_2^2 \cos^2\theta}{2g}$

$$\therefore \frac{H_1}{H_2} = \frac{u_1^2/2g}{u_2^2 \cos^2\theta/2g} = 1 \quad [\text{As } u_1 = u_2 \cos\theta]$$

Short Trick: Maximum height $H \propto T^2$

$$\frac{H_1}{H_2} = \left(\frac{T_1}{T_2}\right)^2$$



EX 3: The ceiling of a long hall is 20 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the wall? (Take g = 10 ms^{-2})

Sol: Here H = 20 m and u = 40 ms^{-1}

Let us assume that the ball is thrown at an angle θ with the horizontal

Maximum height reached (H) $= \dfrac{u^2 \sin^2\theta}{2g}$

$$\Rightarrow \quad 20 = \frac{(40)^2 \sin^2\theta}{2\times 10}$$

$$\Rightarrow \quad \sin\theta = \sqrt{\frac{400}{40\times 40}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 0.5$$

$$\Rightarrow \quad \sin\theta = \sin 30^0 \Rightarrow \theta = 30^0$$

Now Range (R) is given by

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{40^2 \sin\left(2\times 30^0\right)}{10}$$

$$\Rightarrow \quad R = \frac{40\times 40\times 0.866}{10} = 138.56\,m$$

EX 4: A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the same ball? (Take g = 10 ms^{-2})

Sol: Let 'u' be the velocity of projection

Then $R_{max} = \dfrac{u^2}{g} = 100\,m$

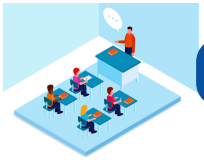
$$\Rightarrow \quad u^2 = 100g \; or \; u = \sqrt{100g}$$

For upward throw of ball, we have

$u = \sqrt{100g}$; v = 0; a = – g

As $v^2 - u^2 = 2as$ $\quad \Rightarrow \quad 0^2 - \left(\sqrt{100g}\right)^2 = 2\times -g\times s$

$$\Rightarrow \quad -100\,g = -2\,gs \qquad \Rightarrow \quad s = \frac{-100}{-2} = 50m$$



CLASSROOM DISCUSSION QUESTIONS

CDQ 06

1. **Two balls of masses m and 2m are thrown upward with the same vertical component of velocity. Which of the following is true?**

(A) Heavier ball goes higher

(B) Lighter ball goes higher

(C) Both reach the same height

(D) Height depends on mass

2. **The formula used to calculate the maximum height H of a projectile projected with speed u at an angle θ is:**

(A) $H = \frac{u^2}{2g}$

(B) $H = \frac{u^2 \sin\theta}{g}$

(C) $H = \frac{u^2 \sin^2\theta}{2g}$

(D) $H = \frac{u^2 \cos^2\theta}{2g}$

3. **Which angle of projection gives the maximum horizontal range for a given speed?**

(A) 30^0 (B) 45^0

(C) 60^0 (D) 90^0

4. **A ball is thrown with a speed of $40 ms^{-1}$ inside a hall of height 20m. The maximum height of the projectile must be:**

(A) Less than 20 m

(B) Equal to 20 m

(C) Greater than 20 m

(D) Independent of ceiling height

5. **Which of the following factors does NOT affect the maximum height of a projectile?**

(A) Initial speed

(B) Angle of projection

(C) Acceleration due to gravity

(D) Mass of the projectile

6. **The time of flight of a projectile depends on:**

(A) Mass of the projectile

(B) Vertical component of velocity

(C) Horizontal component of velocity

(D) Shape of the projectile

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken in Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

C.D.F. CONCEPTS, DEFINITIONS AND FORMULAE

1. A body projected with certain velocity making an angle other than 90° to the horizontal is known as projectile.
2. With respect to the origin, the position of projectile after time 't' is given by x $= u\cos\theta t$ and $y = u\sin\theta t - \frac{1}{2}gt^2$
3. Equation of the projection is

   $y = (\tan\theta)\,x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$ or $y = Ax - Bx^2$
4. Velocity $\vec{V}$ of the body after time 't' is $V_x = u\cos\theta$; $V_y = u\sin\theta\, gt$

   i.e., $\vec{V} = V_x\hat{i} + V_y\hat{j}$; $V = \sqrt{V_x^2 + V_y^2}$

   Angle made by resultant velocity with the horizontal at any instant is $\tan a = \frac{V_y}{V_x}$
5. Maximum height $H = \frac{u^2\sin^2\theta}{2g}$
6. Time of flight $T = \frac{2u\sin\theta}{g}$
7. Horizontal range $R = \frac{u^2\sin 2\theta}{g}$
8. $\tan\theta = \frac{4H}{R}$ or $R = 4H\cot\theta$
9. The horizontal distance travelled by the projectile during the time of flight is called horizontal range.
10. Horizontal range = (Horizontal component of velocity) × (Time of flight)
11. $\frac{U_1}{U_2} = \frac{\sin\theta_1}{\sin\theta_2}$ (For the same H)
12. $\frac{U_1}{U_2} = \sqrt{\frac{\sin 2\theta_2}{\sin 2\theta_1}}$ (For the same R)
13. Horizontal range of the projectile with be maximum if angle of projection $\theta = 45°$.

    Maximum horizontal range $R_{max} = \frac{u^2}{g}$

ADVANCED WORKSHEET

LEVEL 1

Single Correct Answer Type (S.C.A.T)

1. The numerical value of the ratio of average velocity to average speed is _____.

   (A) Always less than one

   (B) Always equal to one

   (C) Always more than one

   (D) Equal to or less than one.

2. A body moves 4m towards east and then 3m north. The displacement and distance covered by the body are:

   (A) 7 m, 6 m (B) 6 m, 5 m

   (C) 5 m, 7 m (D) 4 m, 3 m

3. A body starting from rest moving with uniform acceleration has a displacement of 16m in first 4s and 9m in first 3s. The acceleration of the body is:

   (A) 1 ms^{-2} (B) 2 ms^{-2}

   (C) 3 ms^{-2} (D) 4 ms^{-2}.

4. A particle starts from the origin with a velocity of 10m/s and moves with a constant acceleration till the velocity increases to 50m/s. At that instant, the acceleration is suddenly reversed. What will be the velocity of the particle when it returns to the starting point?

   (A) zero (B) 10m/s

   (C) 50m/s (D) 70m/s

5. When the speed of a car is u, the minimum distance over which it can be stopped is s. If the speed becomes nu, what will be the minimum distance over which it can be stopped during the same time?

   (A) s/n (B) ns

   (C) s/n^2 (D) n^2s.

6. Relation between range (R) and maximum height (H) is given by:

   (A) $Sin\theta = \frac{4H}{R}$

   (B) $Cos\theta = \frac{4H}{R}$

   (C) $Tan\theta = \frac{4H}{R}$

   (D) $Cot\theta = \frac{4H}{R}$

7. Which of the following assumptions to be made in projectile motion?

   (A) There is no resistance due to air

   (B) The effect due to curvature of earth is negligible.

(C) The effect due to rotation of earth is negligible.

(D) All the above

8. **A projectile is fired at an angle of 45° and reached the highest point in its path in $2\sqrt{2}$ s. If air resistance is neglected, maximum height reached by it is:**

(A) g (B) 2g

(C) 3g (D) 4g

9. **The angle of projection for maximum range of projectile is _____.**

(A) 30^0 (B) 45^0

(C) 60^0 (D) 90^0

10. **For the angles of projection, which of the following are same?**

(A) Time of flights

(B) Trajectories

(C) Ranges

(D) Maximum heights

11. **A projectile is fired from a gun with a muzzle velocity of 2000m/s at an elevation of 45°. The maximum height reached would be the same of as that by an identical particle fired vertically with a muzzle velocity of:**

(A) 500 m/s (B) 866 m/s

(C) 1732 m/s (D) 1414 m/s

12. **Two second after projection, a projectile is moving at 30° above the horizontal, after one more second it is moving horizontally. ($g=10ms^{-2}$). The angle of projection is:**

(A) 30° (B) 45°

(C) 60° (D) 90°

13. **A cannon ball has the same range R on a horizontal plane for two different angles of projection. If h_1 and h_2 are the maximum heights in two paths for which this is possible, then:**

(A) $R=h_1h_2$ (B) $R=\sqrt{h_1h_2}$

(C) $R=2\sqrt{h_1h_2}$ (D) $R=4\sqrt{h_1h_2}$

14. **A boy can throw a ball 40m vertically upward. The greatest horizontal distance he can throw the same is:($g = 10\ ms^{-2}$)**

(A) 40m (B) 80m

(C) 60m (D) 120m

15. **The speed of a projectile at its maximum height is 65m/s. If it stays in the air for a total time of 8 seconds, its horizontal range is:**

(A) 12 m (B) 24 m

(C) 36 m (D) 48 m

16. **Two stones are thrown each with a velocity of g from the same point in opposite directions on the ground. If each has the greatest horizontal range, their**

separation on reaching the ground:

(A) g (B) 2g

(C) 3g (D) 4g

17. This horizontal range of a projectile is $R = \frac{gT^2}{2}$ where T is its time of flight. Then angle of projection to the horizontal is:

(A) 30° (B) 45°

(C) 60° (D) 90°

18. The initial velocity of the projectile is equal to the velocity acquired by a freely falling body through a distance 'h'. Then maximum horizontal range of that projectile is:

(A) h (B) 2h

(C) h/2 (D) $\sqrt{2h}$

19. A body is released from the top of a tower of height Hm. After 2s it is stopped and then instantaneously released. What will be its height after next 2s?

(A) (H–5)m (B) (H–10)m

(C) (H–20)m (D) (H–40) m.

Multi Corrrect Questions (M.C.Q)

20. Choose the correct statements from the following. The range of a projectile depends upon:

(A) The angle of projection

(B) The acceleration due to gravity

(C) The magnitude of the velocity of projection

(D) The mass of the projectile

21. A projectile is fired with a constant speed at two different angles of projection, say α and β, that give it the same range. Then α and β are such that:

(A) Cosec α = Sec β

(B) Tan $(\alpha+\beta) \rightarrow \infty$

(C) $\text{Sin}^2\alpha - \text{Cos}^2\alpha = \text{Sin}^2\beta - \text{Cos}^2\beta$

(D) $\text{Cot}\alpha = \text{Cos}\alpha \ \sec\beta$

22. From the top of a tower of height 40m, a ball is projected upwards with a speed of 20ms^{-1} at an angle of elevation of 30°. The total time taken by the ball to hit the ground is T and the time taken to come back to the same elevation is t. The horizontal distance covered by the ball is x. If g = 10 ms^{-2}, then

(A) $\frac{T}{t} = 2$ (B) $\frac{T}{t} = \sqrt{2}$

(C) $x = 40\sqrt{2}$ m (D) $x = 40\sqrt{3}$ m

23. A projectile thrown at an angle of 30° with the horizontal has a range R_1 and attains a maximum

height h_1. Another projectile, thrown with the same speed, at an angle of 30° with the vertical has a range R_2 and attains a maximum height h_2. Then:

(A) $R_2 = 2R_1$ (B) $R_2 = R_1$

(C) $h_2 = 2h_1$ (D) $h_2 = 3h_1$

24. The maximum height attained by a projectile is increased by 1% by increasing its speed of projection without changing the angle of projection. Then the percentage increase in the:

(A) Horizontal range will be 2%

(B) Horizontal range will be 1%

(C) Time of flight will be 0.5%

(D) Time of flight will be 2%

Comprehension Passage (C.P.T.)

Passage - I

Two objects are projected from the same point with the same speed u at angles of projection α and β with the horizontal respectively. They strike the ground at the same point at a distance R from the point of projection. The respective maximum heights attained by the objects are h_1 and h_2 and t_1 and t_2 are the respective times of flight.

25. R, h_1 and h_2 are related as:

(A) $R = \sqrt{h_1 h_2}$

(B) $R = \sqrt{2h_1 h_2}$

(C) $R = 2\sqrt{2h_1 h_2}$

(D) $R = 4\sqrt{h_1 h_2}$

26. The ratio h_1/h_2 is equal to:

(A) $\sin^2\alpha$ (B) $\cos^2\alpha$

(C) $\tan^2\alpha$ (D) $\cot^2\alpha$

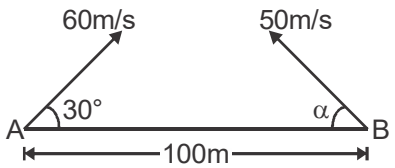
27. The sum $(h_1 + h_2)$ is equal to:

(A) $\frac{u^2}{g}\sin^2\alpha$ (B) $\frac{u^2}{g}\cos^2\alpha$

(C) $\frac{u^2}{g}$ (D) $\frac{u^2}{2g}$

Passage - II

A particle A is projected with an initial velocity of 60m/s at an angle 30° to the horizontal. At the same time a second particle B is projected in opposite direction with initial speed of 50m/s from a point at a distance of 100m from A.



28. If the particles collide in air, find the angle projection a of particle B.

(A) a = $\sin^{-1}$ (3/5)

(B) a = $\sin^{-1}$ ((C)

(C) a = sin (3/5)

(D) $a = \sin^{-1}(5)$

29. If the particles collide in air, find the time when the collisions take place.

(A) 4.2 s (B) 2.1 s

(C) 10.7 s (D) 1.09 s

30. If the particles collide in air, find the distance of P from A. Where collision occurs. ($g = 10\ ms^{-2}$)

(A) 62.64 m (B) 64.62 m

(C) 66.42 m (D) 6645 m

Matrix Matching Type (M.M.T.)

A body is projected with velocity u at angle q = 30° with the horizontal

Column-I

31. Velocity at maximum height

32. Velocity at half the maximum height

33. Average velocity between the point of projection highest point

34. Velocity at $t = \frac{2}{3}$ (time of flight)

Column-II

(A) $\frac{\sqrt{13}u}{4}$ (B) $\frac{\sqrt{7}}{3}u$

(C) $\frac{\sqrt{3}u}{2}$ (D) $\sqrt{\frac{7}{8}}u$

(E) $\sqrt{\frac{8}{15}}u$

Assertion Reason Type (A.R.T.)

(A) Both assertion and reason are true and the reason is the correct explanation of the assertion.

(B) Both assertion and reason are true but reason is not the correct explanation of the assertion.

(C) Assertion is true but reason is false.

(D) Assertion is false but reason is true.

35. Assertion (A): The time of ascent of a projectile is equal to the time of its descent when air resistance is negligible.

Reason (R): Acceleration due to gravity acts equally in magnitude but in opposite direction during ascent and descent.

36. Assertion (A): A body thrown vertically upwards takes less time to go up than to come down.

Reason (R): Acceleration due to gravity is constant.

37. Assertion (A): A ball thrown upward reaches a maximum height where its acceleration becomes zero.

Reason (R): The velocity of the ball becomes zero at the highest

point.

Integer Type Questions (I.T.Q.)

38. What is the horizontal range of the projectile in meters when a body is projected with a speed of 20 m/s at an angle of 30° to the horizontal? ($g = 10\ ms^{-2}$)

39. A projectile is launched at a speed of 50 m/s at an angle of 45° with the horizontal. Find the total time of flight. ($g = 10\ ms^{-2}$)

40. A body is dropped from a height of 80 meters. How long does it take to reach the ground? ($g = 10\ ms^{-2}$)

Previous Questions (P.Q.)

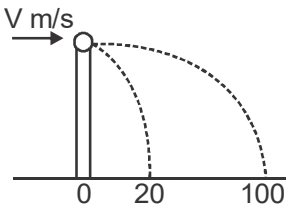
41. A body falling freely from a given height H hits an inclined plane in its path at a height h. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/H) the body will take maximum time to reach the ground?

[iit-jee:1986]

(A) 0.25s (B) 0.5s

(C) 0.75s (D) 1s

42. 1 A ball A ball of mass 0.2kg rests on a vertical of heights 5m. A bullet of mass 0.01kg, travelling with a velocity Vm/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. They hits the ground at a distance of 20m and the bullet at a distance of 100m from the foot of the post. The initial velocity V of the bullet is: [IIT-JEE: 2011]

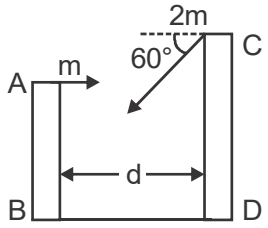


(A) 250 m/s (B) $250\sqrt{2}$ m/s

(C) 400 m/s (D) 500 m/s

43. Two towers AB and CD are situated a distance d apart as shown in figure. AB is 20m high and CD is 30m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10m/s towards CD. Simultaneously another object of mass 2m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other. Calculate the distance d between the towers.

[IIT - JEE: 1994]



(A) 16.45 m (B) 17.32 m

(C) 18.45.m (D) 19.32 m

44. Two guns, situated on the top of a hill of height 10m, fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find

(A) the time interval between the firings and

(B) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the nuzzle and trajectories in X-Y plane.

[IIT-JEE: 1996]

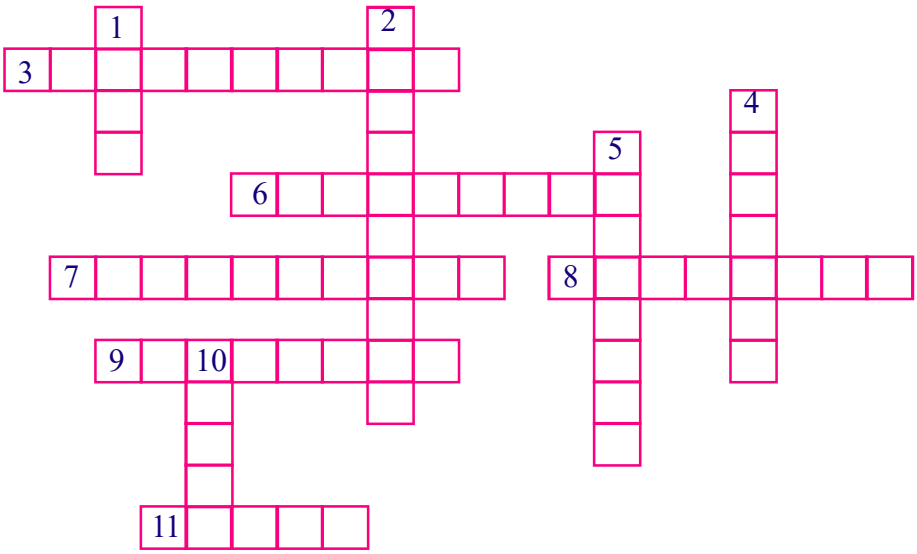
(A) 1sec, $(5\sqrt{3},5)$

(B) 1sec, $(5\sqrt{3},3)$

(C) 2sec, $(3\sqrt{3},4)$

(D) 2sec, $(4\sqrt{2},4)$

Puzzle Time



ACROSS (→)

3. The _____ of a projectile is a parabola.
6. The ground on which the projectile is thrown is taken as _____ level.
7. For a projectile, the velocity in _____ direction remains constant.
8. The path of projectile as observed from other projectile is a _____ line.
9. The path of projectile is a _____.
11. The acceleration of a projectile in the horizontal direction is _____

DOWN (↓)

1. A projectile has _____ range for angle of projections q and $(90^0 - q)$.
2. A body projected with certain velocity at an angle other than 90^0 to the horizontal is known as _____
4. The time of descent is independent of _____ velocity.
5. The acceleration of a projectile in _____ direction is 'g'.